Department of Mechanical Engineering

ME EN 7960 - Precision Machine Design

Syllabus

Instructor:	Prof. Eberhard Bamberg, 2124 MEB, bamberg@mech.utah.edu
Office hours:	Wednesday, 2:15 - 3:45 pm
Lectures:	Monday and Wednesday, 12:55-1:45 pm, MEB 1225
Course website:	http://www.mech.utah.edu/~me7960
Recommended text:	Class notes available for download from course website *Precision Machine Design*. Alexander H. Slocum, Society of Manufacturing Engineers, Dearborn, MI, 1995
Prerequisites:	ME EN 3910 or similar machine elements course, instructor's permission

Course Summary

The course provides an intensive coverage of precision engineering theory and its application to the design of complex production equipment such as machine tools, robots, and metrology devices.

Topics covered include: precision design philosophy; the current state of art; principles of accuracy, repeatability, and resolution; error budgeting; sensors and sensor mounting; contact and non-contact bearings for rotary and linear motion; actuators and transmissions; structural design; dynamic response and structural damping; solid modeling of complex systems.

The course aims to help students in developing creative designs and teaches techniques that allow the designs to be optimized through analytical and numerical methods. Weekly homework assignments provide ample opportunities to practice the concepts learned. The course culminates in a final project.

Final Project

The final project is designed as an exercise that allows the material learned in this class to be applied to a real world problem. The instructor will propose a limited number of projects to choose from. All projects will need to include the following:

- list of functional requirements
- error budget
- proof of design optimization (finite element analysis, analytical approaches, etc.)
- bill of materials
- complete solid model (assembly)
- engineering drawings

The project and some of the homework assignments require students to use a 3D solid modeling package which students are free to choose. This course does not teach any particular design or analysis software.

Grading

Assignments - 30%

Midterms - 30%

Final Project - 40%

TABLE 1 Course schedule Fall 2006

Week	Monday	Wednesday
Week 1 (08/21 - 08/25)		Course Introduction / Examples of PMDs
Week 2 (08/28 - 09/01)	Fundamentals of Errors	Fundamentals of Errors
Week 3 (09/04 - 09/08)	*Labor Day - no classes*	Linear Motion Systems
Week 4 (09/11 - 09/15)	Linear Motion Systems	Ball Screw Calculations
Week 5 (09/18 - 09/22)	Ball Screw Calculations	**1. Midterm**
Week 6 (09/25 - 09/29)	Alternative Linear Motion Systems	Alternative Linear Motion Systems
Week 7 (10/02 - 10/06)	Sensors and Encoders	Sensors and Encoders
Week 8 (10/09 - 10/13)	Contact Stresses and Deformations	Contact Stresses and Deformations
Week 9 (10/16 - 10/20)	Linear Rolling Element Bearing	**Take Home Midterm**
Week 10 (10/23 - 10/27)	*no class*	Rotary Rolling Element Bearings
Week 11 (10/30 - 11/03)	Non-Contact Bearings	Modeling Bearings
Week 12 (11/06 - 11/10)	Modeling Bearings	Flexures
Week 13 (11/13 - 11/17)	Design of Machine Structures	Design of Machine Structures
Week 14 (11/20 - 11/24)	Modal Analysis	Modal Analysis
Week 15 (11/27 - 12/01)	Project Discussions	Project Discussions
Week 16 (12/04 - 12/08)	Project Discussions	**Final Projects due (5pm in MEB 2110)**

Examples of Precision Machine Design

ME EN 7960 – Precision Machine Design
Topic 1

Definition

- A precision machine device is defined as a piece of equipment with one or more moving parts whose accuracy determines the quality of the device
- Examples of precision machine devices include:
  - Machine Tools
    - Milling Machines
    - Lathes
    - Grinders
    - Laser Cutter
    - WaterJets
    - Non-traditional machines (EDM, ECM, …)
  - Coordinate Measuring Machines
  - Production Equipment
    - Presses
  - Mechanical watches
  - Etc…

Traditional Machine Tools – Vertical Machining Center

Source: Mazak

Source: Precision Design Lab

- 3 linear axes (typical)
- 1-5 rotary axes (optional)
- 1-30 kW spindle power (typical)
- 0-10,000 rpm spindle speed (typical)
- > $50,000

3D parts (non-symmetrical)

Traditional Machine Tools - Lathe

Source: Hardinge

- 2 linear axes + 1 rotary axis (typical)
- 1-2 rotary axes (optional)
- 1-15 kW spindle power (typical)
- 0-6,000 rpm spindle speed (typical)
- > $25,000

3D parts (rotationally symmetric)

3D parts (rotationally symmetric) with off-axis features (requires 2nd spindle)

Traditional Machine Tools – Surface Grinder

Source: Jung GmbH

Source: THK

2D parts (flat or profiled)

- 3 linear axes + 1 rotary axis (typical)
- 1- 2 rotary axes (optional)
- 5-15 kW spindle power (typical)
- 800-4,000 rpm spindle speed (typical)
- > $20,000

Traditional Machine Tools – Circular Grinder

Source: Studer GmbH

Source: Crowder Supply

3D parts (rotationally symmetric)

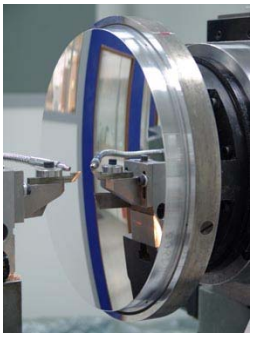
- 2 linear axes + 2 rotary axes (typical)
- 5-10 kW spindle power (typical)
- 800-4,000 rpm spindle speed (typical)
- > $25,000

New Machine Tools – Diamond Turning

Source: Precitech

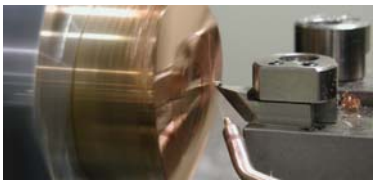
- 2 linear axes (typical)
- 1-15 kW spindle power (typical)
- 0-7,500 rpm spindle speed (typical)
- > $150,000

3D parts (rotationally symmetric)



Source: Savimex

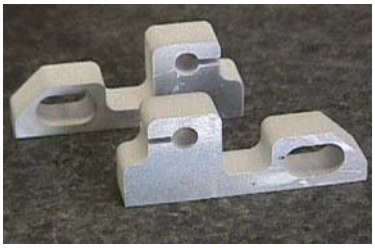
3D parts (asymmetric) with fast tool servo (optional)

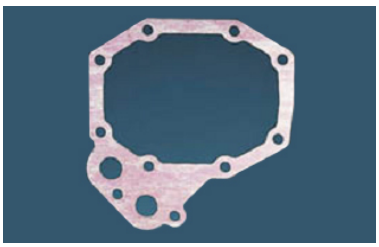


Source: Precitech

New Machine Tools - WaterJet

Source: OMAX

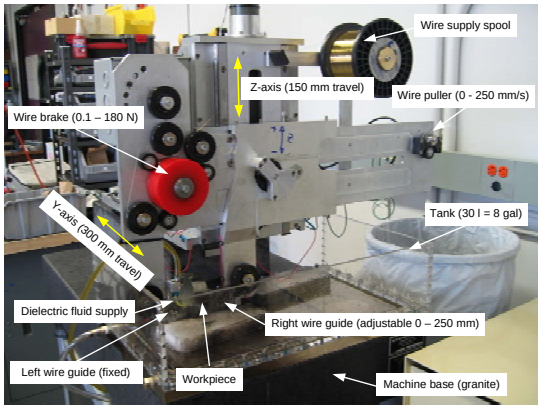




- 2 linear axes (typical)
- Pump power (15-40 kW)
- > $20,000

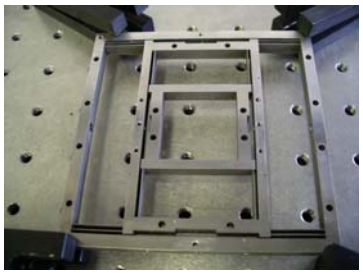
2D parts (from flat stock material)

New Machine Tools – Wire –Electro Discharge Machine



Source: Precision Design Lab

2D parts from flat stock

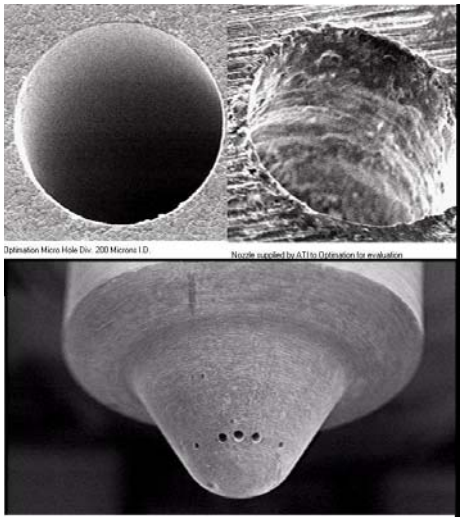


Source: Precision Design Lab

- 2 linear axes (typical)
- 4 linear + rotary axes (optional)
- 1-10 kW power supply (typical)
- > $100,000

New Machine Tools – Sinker Electro-Discharge Machine

Source: Charmilles



Source: Optimation

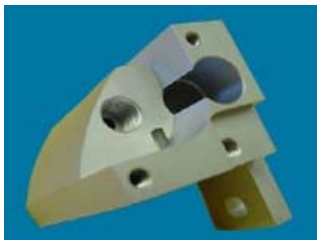
- 2D or 3D parts (from flat stock material)
- 3 linear axes (typical)
- 0-5 kW power supply
- > $80,000

New Machine Tools – Electro Chemical Machining

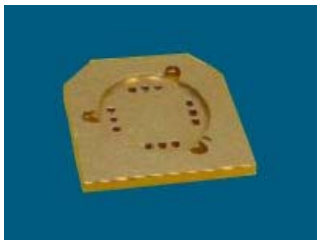
Source: Ultra Systems Ltd.

- 1 linear axes (typical)
- 3 linear + rotary axes (optional)
- 1-10 kW power supply (typical)
- > $100,000

3D parts



2D parts



Coordinate Measuring Systems

Specifications

Model No.		QM-Measure 333	QM-Measure 353
Measuring range	X-axis	12" (300mm)	12" (300mm)
	Y-axis	12" (300mm)	20" (500mm)
	Z-axis	12" (300mm)	12" (300mm)
Length standard		Precision linear encoder	
Resolution		.00002" (0.0005mm)	
Accuracy (20°C ± 1°C)*		E = (3.0 + 4L/1000)µm R = 4.0µm (.00016")	
Guide method		Air bearing for each axis	
Clamping method		Clamping screw	
Fine feeding device		Optional (.4" / 10mm stroke)	
Z-axis balance		Counterweight	
Measuring table		Granite sub-plate	
Machine stand		Optional	
Workpiece loading		Maximum height: 16.14" (410mm) Maximum mass: 66 lbs. (30kg)	
Air Pressure		51PSI or 0.35MPa	
Air consumption		50L/min. (in normal state) or 1.8CFM	
Dimensions (QM-Measure)	Width	32.48" (825mm)	32.48" (825mm)
	Depth	27.17" (690mm)	35.04" (890mm)
	Height	52.95" (1345mm)	53.54" (1360mm)
Mass**	QM-Measure	286 lbs. (130 kg)	374 lbs. (170 kg)
	QM-Data 300	2.6 lbs. (1.2 kg)	2.6 lbs. (1.2 kg)

Source: Mitutoyo

Performance Criteria

Precision Devices are rated according to:

- Manufacturing Capability
  - Accuracy
  - Resolution
  - Repeatability
- Ability to provide motion
  - Number of axes (complexity of machined geometry)
  - Speed (determines the manufacturing time)
  - Travel (determines the part size)
- Ability to provide energy required for manufacturing processes
  - Spindle power (machining centers, lathe, grinder, etc.)
  - Laser power (laser cutters)
  - Pump pressure (WaterJet)
  - Voltage, Current, Frequency (EDM, ECM)

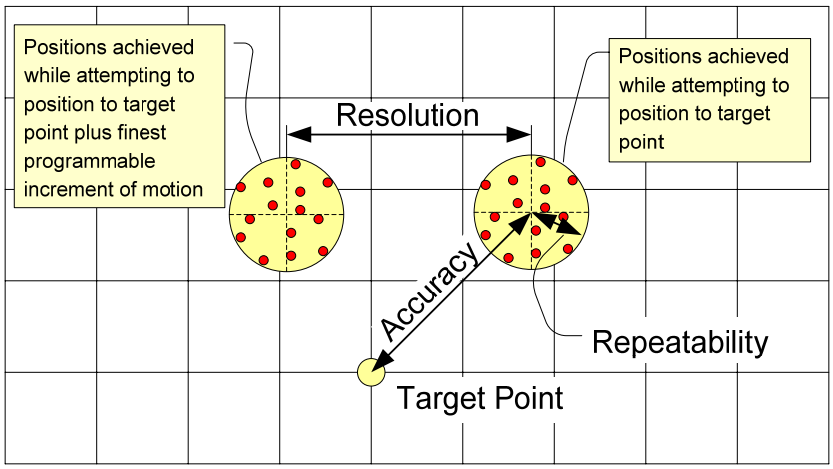
Performance Limiting Criteria

- Most machines create geometry by selectively removing material.
- The amount and location of the material removed is a function of the amount and location of the energy transmitted from the tool (machine) to the work piece.
- The location of the energy transmission is rarely controlled directly but indirectly at remote measurement points.
- As a result, the created geometry will always deviate from the perfect shape.
- The amount of this deviation can be defined as the achievable precision of both the machine and the process.

Fundamentals of Errors

ME EN 7960 – Precision Machine Design
Topic 2

Accuracy, Repeatability, and Resolution



Accuracy

- Accuracy is the ability to tell the truth or:
  - The maximum translational or rotational error between any two points in the machine's work volume
  - Linear, planar, and volumetric accuracy can all be defined for a machine
- Note: mechanical accuracy is far more costly than repeatability

Repeatability

- Repeatability (precision) is the ability to tell the same story over and over again or:
  - The error between a number of successive attempts to move the machine to the same position
  - Repeatability is often considered to be the most important parameter of a computer controlled machine (or sensor)
  - Often the intent is to map the errors and then compensate for them
- Minimize static friction, backlash, and thermal variants to get better repeatability

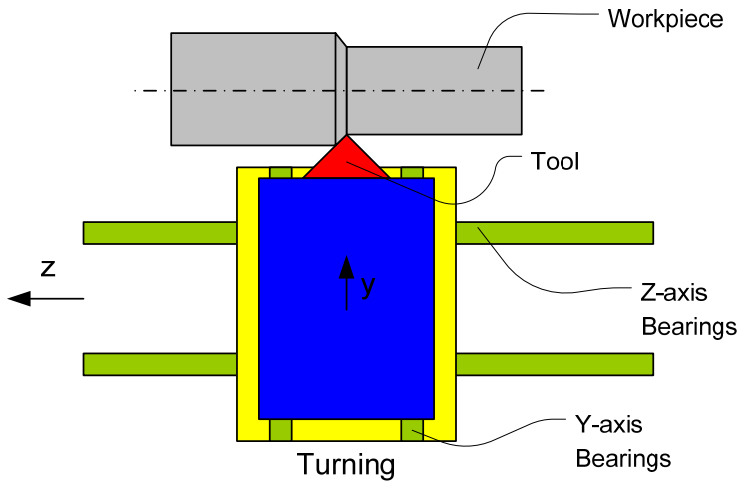
Resolution

- Resolution is how detailed your story is
  - Resolution is the larger of the smallest programmable step or:
  - The smallest mechanical step the machine can make during point to point motion
  - Resolution gives a lower bound on the repeatability
- Minimize static friction and use higher resolution feedback to get better resolution

What is the importance of errors?

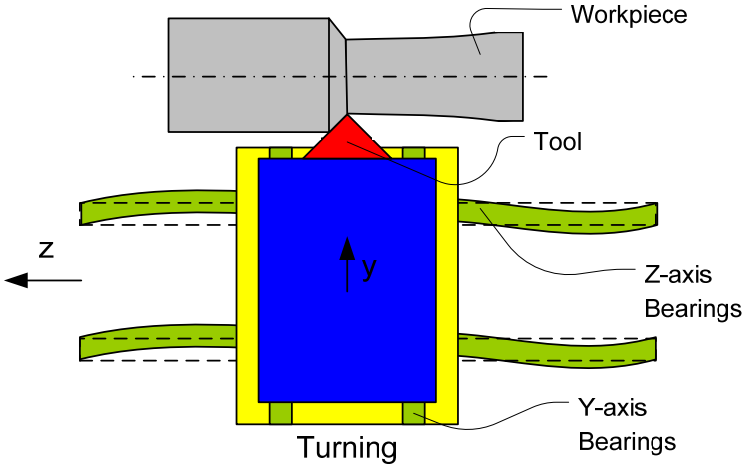
- The trajectory of the tool relative to the work piece directly determines the dimensions of all features machined
- No machine tool monitors the tool trajectory directly. Instead, the tool trajectory is monitored indirectly by monitoring the position of the moving components (axes)
- Any deviation of any moving component will imprinted onto the work piece, thereby reducing the part accuracy

Imprinting Form Errors



Case 1: Perfectly straight z-axis bearings create a perfectly cylindrical part

Imprinting Form Errors (contd.)



Case 2: The straightness errors of the z-axis bearings are transferred onto the part

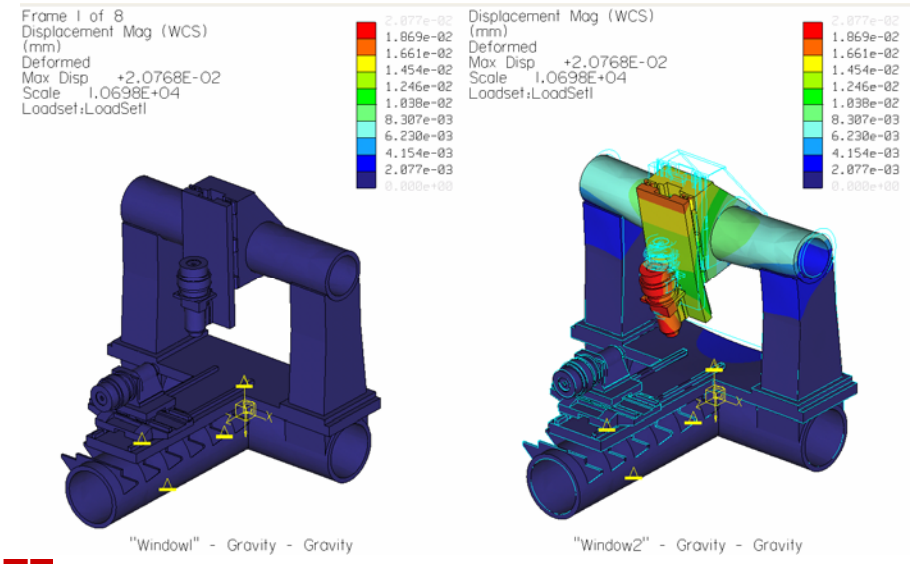
Performance Limiting Criteria

- Accuracy and Repeatability are limited by:
  - Geometric errors of all components
  - Kinematic errors
  - Load induced errors
  - Thermal errors
  - Dynamic errors
  - Calibration errors
  - Computational errors
- Resolution is limited by
  - Quality of sensors
  - Quality of control system
  - Friction (stick and slip effect)
  - Backlash

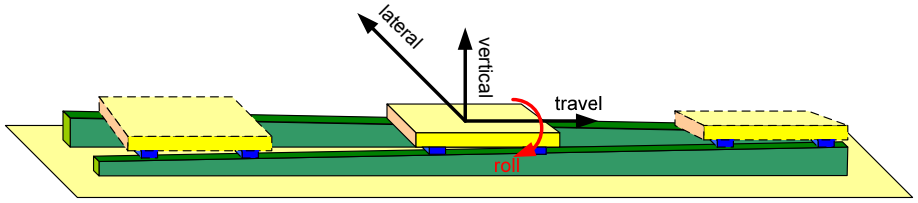
Geometric Errors

- Errors in the form of individual machine components:
  - Component straightness error due to machining errors
  - Component straightness error due to gravity loading
- Surface finish effects
- Quasi-static accuracy of surfaces moving relative to each other (e.g., linear or rotary motion axes):
  - Linear motion axis:
    - Pitch
    - Roll
    - Yaw
    - Straightness (2 components)
    - Linear displacement

Geometric Error - Gravity

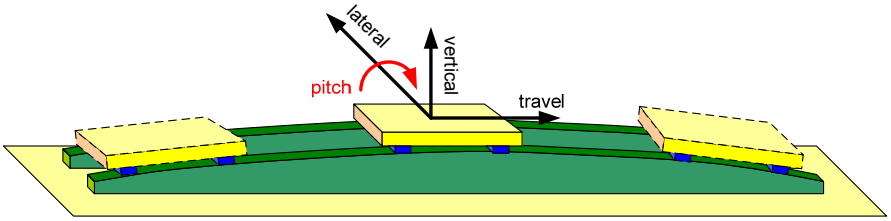


Geometric Errors - Roll



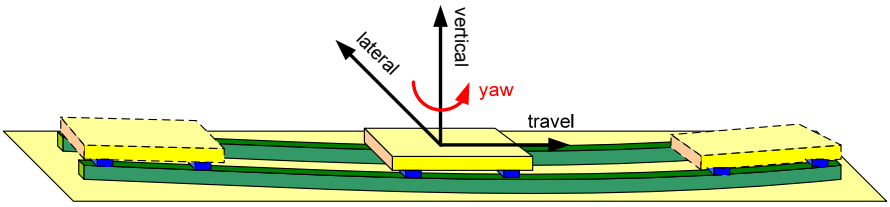
Roll as a result of different slopes of the rails

Geometric Errors - Pitch



Pitch as a result of a bow of the rails in the vertical direction

Geometric Errors - Yaw

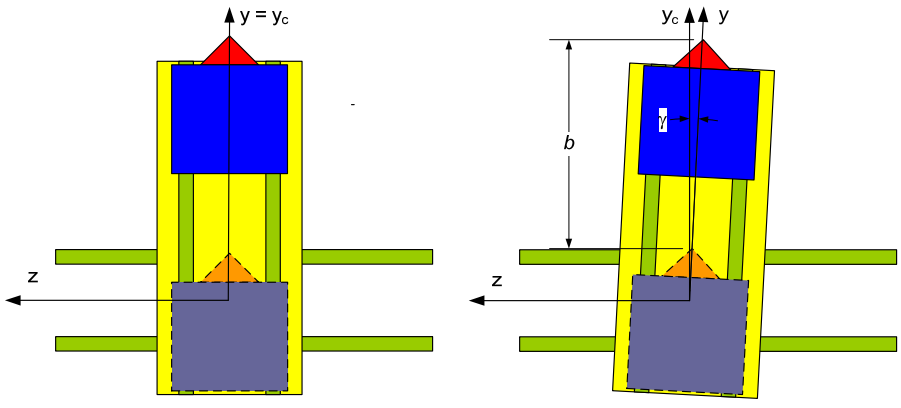


Yaw as a result of bow of the rails in the lateral direction

Other Errors

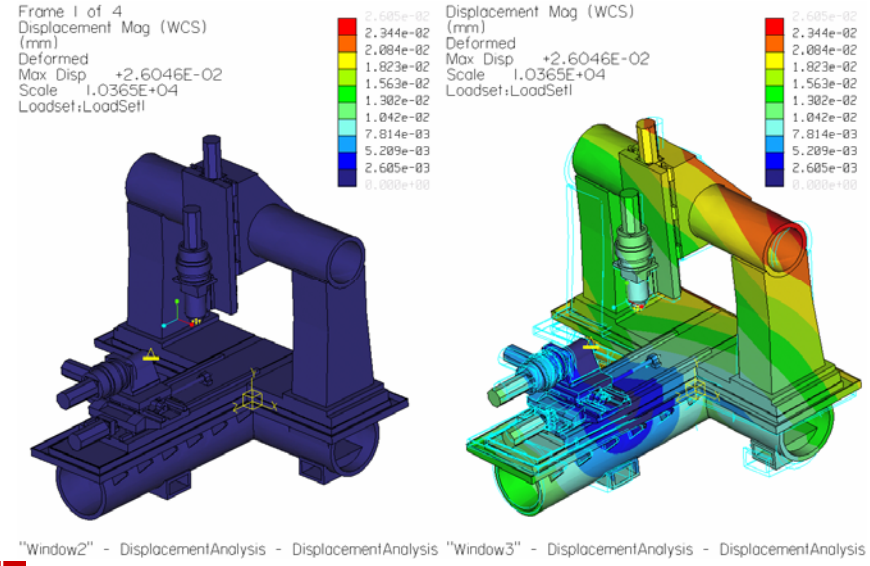
- Kinematic Errors
  - Errors in an axis's trajectory that are caused by misaligned or improperly sized components
    - Orthogonality between axes
    - Parallelism between axes
    - Error motions in a closed kinematic chain
- External load induced errors:
  - Errors due to deformation of components:
    - Gravity load induced errors
    - Cutting/probing force induced errors
    - Axis acceleration load induced errors

Kinematic Error - Orthogonality Between Axes



$$\Delta z = b \cdot \gamma$$

Geometric Error – Load Induced



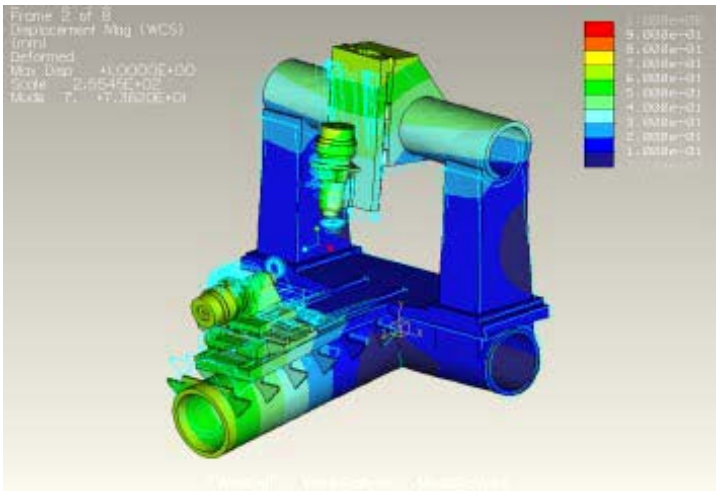
Thermal Errors

- Mean temperature other than 68 °F (20 °C)
  - Gradients in environment's temperature
  - Errors caused by thermal expansion of elements:
    - External heat sources:
      - Mean temperature of the room
      - Sun shining through the window onto the machine
      - Nearby machine's hot air vent
      - Overhead lights
      - Operator's body heat
    - Internal heat sources:
      - Motors
      - Bearings
      - Machining process
      - Pumps
      - Expansion of compressed fluids
      - Coolant

Dynamic Errors

- Errors caused by vibration or control processes:
  - Vibration:
    - External environment (usually through the ground)
    - Cutting process
    - Rotating masses
  - Control system:
    - Algorithm type (e.g., PID, adaptive, etc.)
    - Stick-slip friction
    - Varying mass
    - Varying stiffness
  - Switching amplifiers
    - Servo loop frequency excites a natural mode of the machine

Dynamic Errors



Calibration Errors

- Errors associated with sensors:
  - Intrinsic accuracy
  - Interpolation
  - Mounting errors:
    - Position
    - Mounting stress
  - Calibration (error associated with the mastering process)

Additional Errors

- Computational errors:
  - Error introduced in the analysis algorithms
  - Rounding off errors due to hardware
- Additional sources of error (often very difficult to model):
  - Humidity
  - Loose joints
  - Dirt
- Variations in supply systems:
  - Electricity
  - Fluid pressure
  - Operator inattention
  - Fluid supply cleanliness
- Operators (“52-2” factor)

Error Assessment and Budgeting

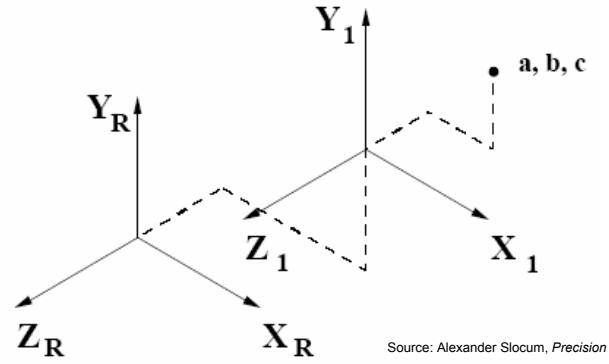
- Given all the different types of errors that can affect all different components:
  - Keeping track of all the errors is such a daunting task:
    - Most engineers don't bother and use "experience" to guide the design
    - It is left up to manufacturing and service to work the bugs out
  - This seems to be a major source of reliability and performance problems
- The solution to a successful project is a good budget:
  - A project requires a good **financial budget** to make it feasible
  - A project requires a good **time budget** to make it feasible
  - A project requires a good **error budget** to make it feasible
- In order to make a good error budget for the system, a good mathematical model is needed

Homogenous Transformation Matrices

- Allows the designer to consider one part of the machine at a time, and then link them all together like beads on a string
- Based on rigid body model of a linear series (open chain) of coordinate frames
- Takes into account linear and angular offsets between coordinate frames
- Transforms XYZ coordinates of one frame into XYZ coordinates of another frame
- Used to transform locally referenced errors into errors referenced with respect to the tool point and the work piece

Homogenous Transformation Matrices (contd.)

- Coordinate frames are placed at bearings, joints, and areas where other parameters are lumped
- Closed chains (e.g. a five point bearing mount) need to be modeled with the generation of constraint equations



Source: Alexander Slocum, *Precision Machine Design*

Structure of HTM's

Orientation of Y_n with respect to adjacent coordinate frame

Orientation of X_n with respect to adjacent coordinate frame

Orientation of Z_n with respect to adjacent coordinate frame

$$R_{T_n} = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & P_x \\ O_{jx} & O_{jy} & O_{jz} & P_y \\ O_{kx} & O_{ky} & O_{kz} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of $X_nY_nZ_n$ with respect to adjacent coordinate frame

Structure of HTM's (contd.)

- The first three columns are the direction cosines (unit vectors i, j, k).
  - They represent the orientation of the X_n, Y_n, and Z_n axes with respect to an adjacent coordinate frame
- The last column is the position of the rigid body's coordinate system's origin with respect to the reference frame
- The pre-superscript represents the reference frame in which you want the result to be represented
- The post-subscript represents the reference frame from which you are transferring:

Structure of HTM's (contd.)

- The equivalent coordinates of a point in a reference frame n, in a reference frame R are:

Equivalent coordinates of a point in reference frame *n*, in reference frame *R*

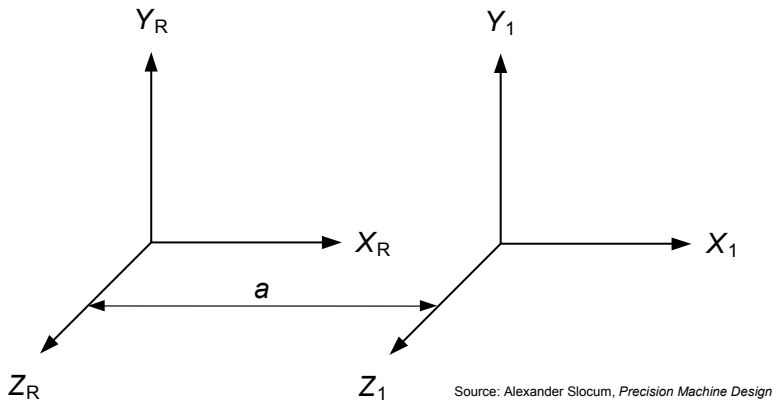
Equivalent coordinates of a point in reference frame *n*, in reference frame *R*

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \\ 1 \end{bmatrix} = R_{T_n} \begin{bmatrix} X_n \\ Y_n \\ Z_n \\ 1 \end{bmatrix}$$

HTM that describes the transformation of reference frame *n* into reference frame *R*

Example: Translation in X

- The coordinate system $X_1Y_1Z_1$ is shifted in X by an amount a:



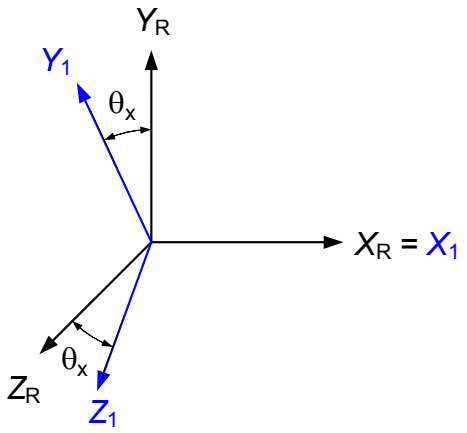
Source: Alexander Slocum, *Precision Machine Design*

HTM of Translation in x

$$XYZ_{T_{X_1Y_1Z_1}} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example: Rotation about X-Axis

- the $X_1Y_1Z_1$ coordinate system is rotated by an amount θ_X about the X axis:



HTM of Rotation about X

$$XYZ_{T_{X_1Y_1Z_1}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Sequential Systems

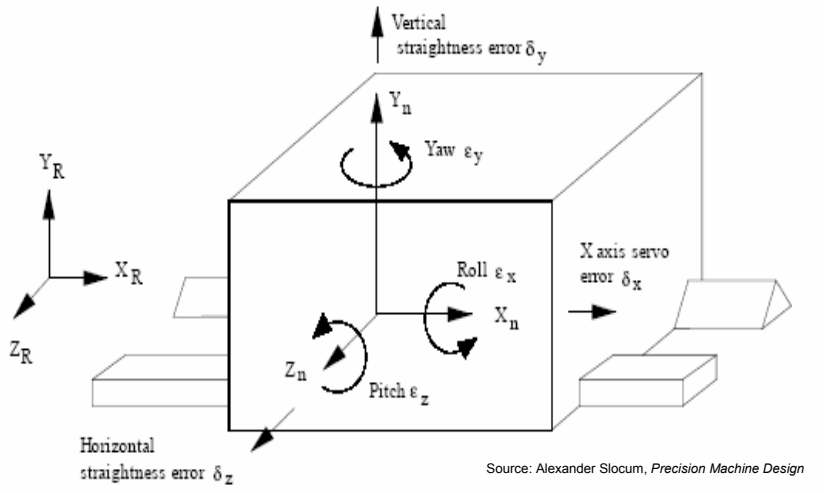
- Transformation from the N[th] axis to the reference system will be the sequential product of all the HTMs:

$$R_{T_N} = \prod_{m=1}^{N} {}^{m-1}T_m = {}^{0}T_1\,{}^{1}T_2\,{}^{2}T_3 \dots$$

- The error between the tool and the work piece is obtained from:

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}_{work,R} - \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}_{tool,R}$$

Rigid body models of machine components

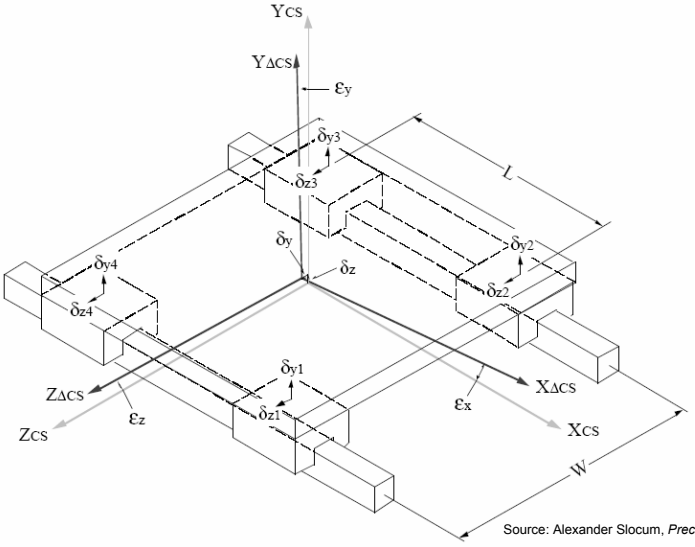


Source: Alexander Slocum, *Precision Machine Design*

HTM of Linear Motion Carriage

$$R_{T_{nerr}} = \begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & a+\delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & b+\delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & c+\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Estimating Position Errors from Catalogs

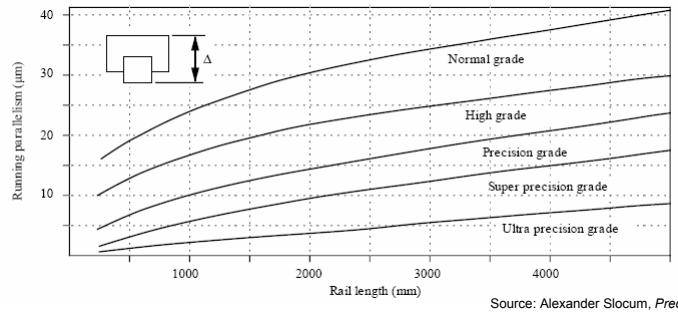


Source: Alexander Slocum, *Precision Machine Design*

HTM of Modular Bearing System

$$\begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & a+\delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & b+\delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & c+\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HTM of Modular Bearing System (contd.)



Source: Alexander Slocum, *Precision Machine Design*

The translational errors are based on the average of the straightness errors experienced by the bearing block:

$$\delta_{axial} = \delta_{servo}$$

$$\delta_{lateral} = \frac{\delta_{lateral1} + \delta_{lateral2} + \delta_{lateral3} + \delta_{lateral4}}{4}$$

$$\delta_{vertical} = \frac{\delta_{vertical1} + \delta_{vertical2} + \delta_{vertical3} + \delta_{vertical4}}{4}$$

HTM of Modular Bearing System (contd.)

- The angular errors are based on the differences in the average straightness errors experienced by pairs of bearing blocks acting across the carriage:

$$\varepsilon_{axial} = \frac{\frac{\delta_{vertical2} + \delta_{vertical3}}{2} - \frac{\delta_{vertical1} + \delta_{vertical4}}{2}}{W}$$

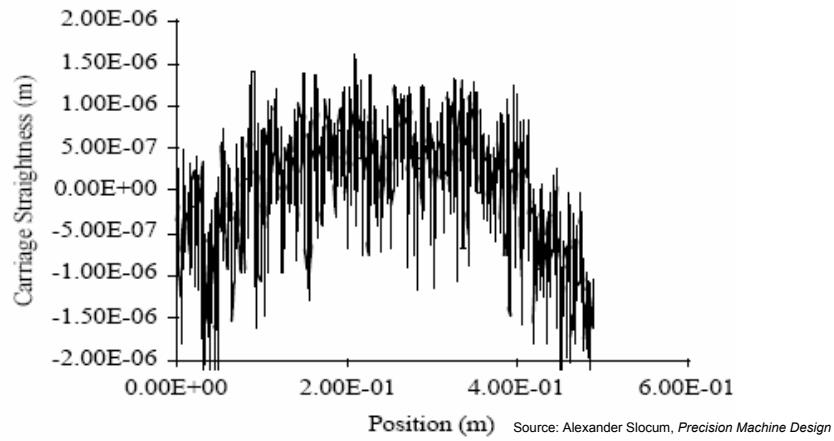
$$\varepsilon_{lateral} = \frac{\frac{\delta_{vertical1} + \delta_{vertical2}}{2} - \frac{\delta_{vertical3} + \delta_{vertical4}}{2}}{L}$$

$$\varepsilon_{vertical} = \frac{\frac{\delta_{lateral3} + \delta_{lateral4}}{2} - \frac{\delta_{lateral1} + \delta_{lateral2}}{2}}{L}$$

HTM of Modular Bearing System (contd.)

- Note we assumed for the straightness we assumed all the errors are acting in the same direction
- Here we assume one set of errors acts up, and the other acts down
- This is very conservative (makes up for other effects we might miss)

Linear Error Plot

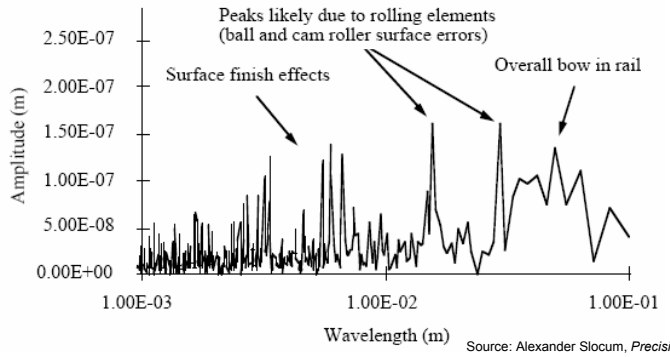


Source: Alexander Slocum, *Precision Machine Design*

Straightness error of a kinematically (statically determined) supported linear axis (5 rolling element bearings on a vee and flat)

Fourier Transformation of Linear Error Plot

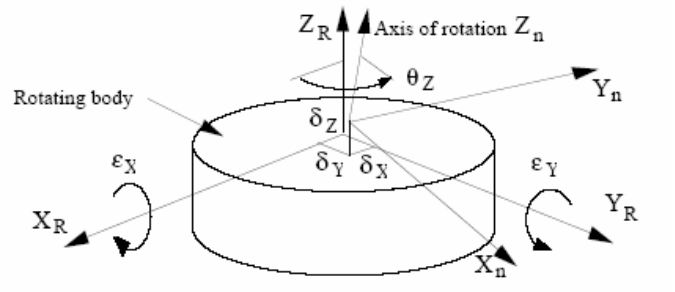
- The Fourier Transformations when plotted as the error amplitude as a function of wavelength is an invaluable diagnostic tool



Source: Alexander Slocum, *Precision Machine Design*

- The wavelength of rolling elements is $2\pi D$

HTM of Rotary Axis



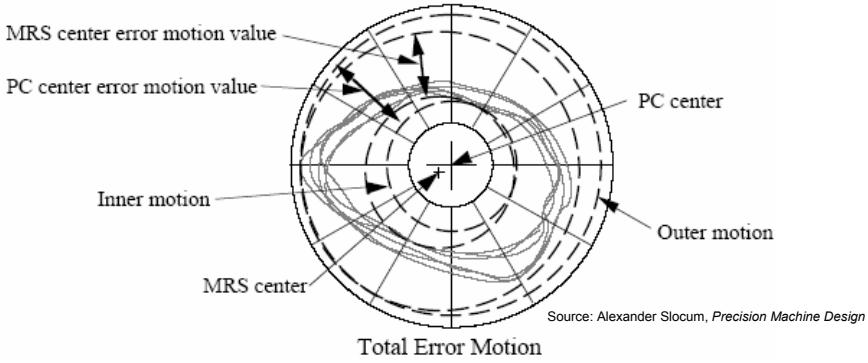
Source: Alexander Slocum, *Precision Machine Design*

$$R_{T_{nerr}} = \begin{bmatrix} \cos\varepsilon_y \cos\theta_z & -\cos\varepsilon_y \sin\theta_z & \sin\varepsilon_y & \delta_x \\ \sin\varepsilon_x \sin\varepsilon_y \cos\theta_z + \cos\varepsilon_x \sin\theta_z & \cos\varepsilon_x \cos\theta_z - \sin\varepsilon_x \sin\varepsilon_y \sin\theta_z & -\sin\varepsilon_x \cos\varepsilon_y & \delta_y \\ -\cos\varepsilon_x \sin\varepsilon_y \cos\theta_z + \sin\varepsilon_x \sin\theta_z & \sin\varepsilon_x \cos\theta_z + \cos\varepsilon_x \sin\varepsilon_y \sin\theta_z & \cos\varepsilon_x \cos\varepsilon_y & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotary Errors

- This general result may also be used for the case of a linear motion carriage if ε_z is substituted for θ_z
- Radial error motion is conservatively estimated to be the ABEC bore tolerance
- Tilt error motion is conservatively estimated to be the ABEC axial tolerance divided by the bearing OD

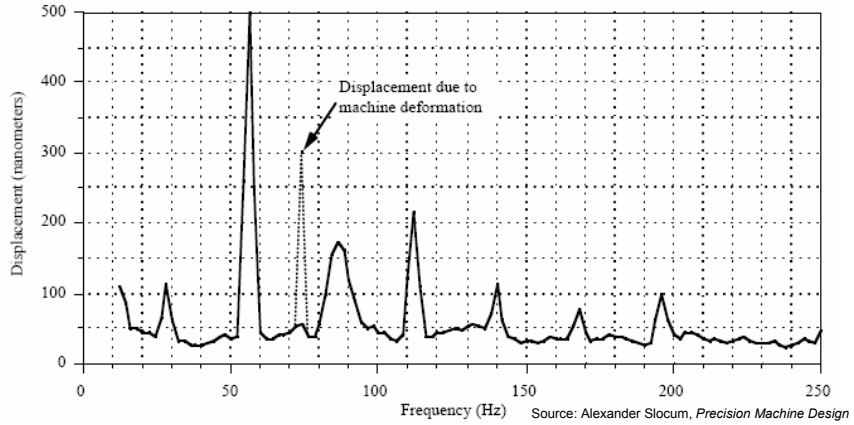
Rotary Error Plot



Source: Alexander Slocum, *Precision Machine Design*

- The average error motion is indicative of the form error that will be imparted to the part when held in a lathe spindle
- The asynchronous error motion is indicative of the surface finish that will be obtained

Fourier Transformation of Radial Error



Source: Alexander Slocum, *Precision Machine Design*

- Spindle speed: 1680 rpm (28 Hz)
- Bearing ID: 75 mm
- Bearing OD: 105 mm
- Number of balls: 20
- Ball diameter: 10 mm
- Contact angle: 15

Combinational Ruler for 3 Common Types

- **Random** - under apparently equal conditions at a given position, errors that do not always have the same value, and can only be expressed statistically
- **Systematic** - which always have the same value and sign at a given position and under given circumstances
  - Generally can be correlated with position along an axis and can be corrected
    - If the relative accompanying random error is small enough
- **Hysteresis** - a systematic error which in this instance is separated out for convenience
  - Usually repeatable, sign depends on the direction of approach, and magnitude partly dependent on the travel
  - May be compensated for if the direction of approach is known and an adequate pre-travel is made

Linear Motion Systems

ME EN 7960 – Precision Machine Design
Topic 3

Major Topics

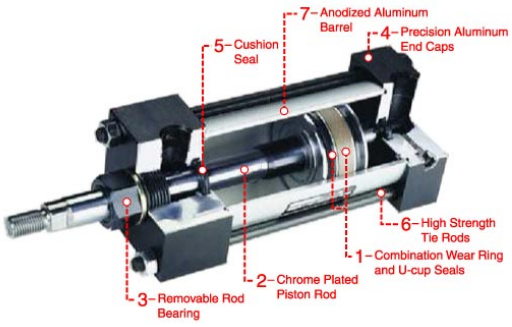
- Error sources
- Pneumatic and hydraulic cylinders
- Belt drives
- Rack and pinion drives
- Friction drives
- Lead screws
- Linear electric motors

Error Sources

- There are five principal error sources that affect linear actuator performance:
  1. Form error in the device components
  2. Component misalignment
  3. Backlash
  4. Friction
  5. Thermal effects
- These systems often have long shafts (e.g. ball screws)
  - One must be careful of bending frequencies being excited by rotating motors

Pneumatic Cylinders

- Compressed gas enters one side of the pressure chamber
- Displacement of piston occurs until force equilibrium is established
- Stick-slip occurs due to piston seals
- Gas is compressible, making motion control very difficult



Source: Milwaukee Cylinder

→ Not useful for precision motion control

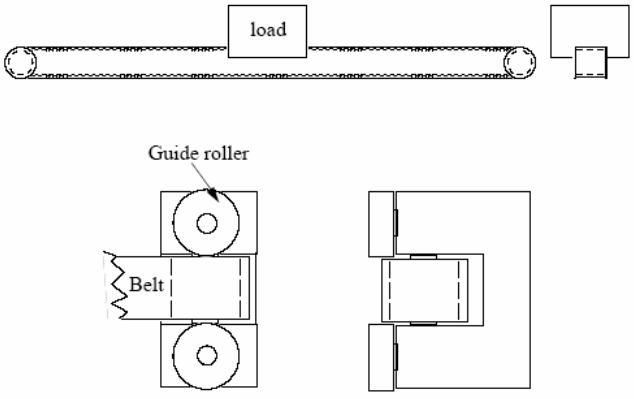
Hydraulic Cylinders

- Compressed oil enters one side of the pressure chamber
- Displacement of piston occurs until force equilibrium is established
- Stick-slip occurs due to piston seals.
- Oil is virtually incompressible, making motion control somewhat doable.

→ Not useful for precision motion control

Belt Drives

- To prevent the belts' edges wearing on pulley flanges:
  - Use side rollers to guide timing belt to prevent wear caused by flanged sheaves:



Source: Alexander Slocum, *Precision Machine Design*

Belt Drives (contd.)

- Used in printers, semiconductor automated material handling systems, robots, etc.
  - Timing belts will not slip
  - Metal belts have greater stiffness, but stress limits life

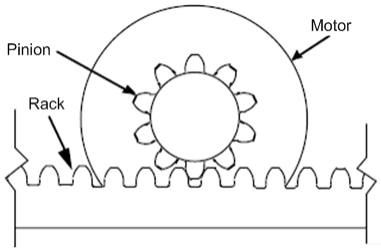
$$\sigma = \frac{Et}{2\rho}$$

E Young's modulus
t Belt thickness
ρ Bending radius

- Timing belts will be the actuator of choice for low cost, low, stiffness, low force linear motion until:
  - Linear electric motor cost comes down
  - PC based control boards with self-tuning modular algorithms become more prevalent

→ Not useful for precision motion control

Rack and Pinion Drives



Source: Alexander Slocum, *Precision Machine Design*

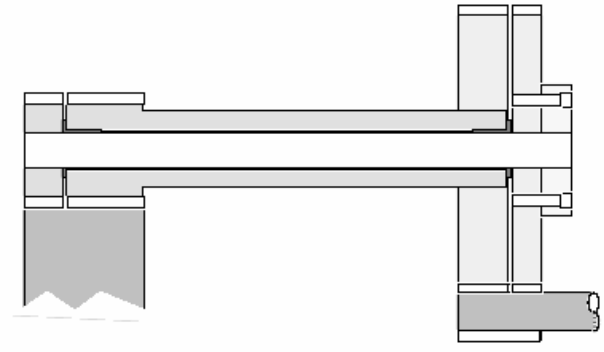
- One of the least expensive methods of generating linear motion from rotary motion
  - Racks can be placed end to end for as great a distance as long as one can provide a secure base on which to bolt them

Rack and Pinion Drives (contd.)

- Commonly used on very large machines such as gantry robots and machining centers used in the aircraft industry
- It is difficult to obtain the "optimal transmission ratio"
  - A speed reducer is sometimes used with the motor that drives the pinion
  - They do not provide a mechanical advantage the way a lead screw system does
- The characteristics of gears apply here equally well, including the use of anti-backlash or multiple pinions

Rack and Pinion Drives (contd.)

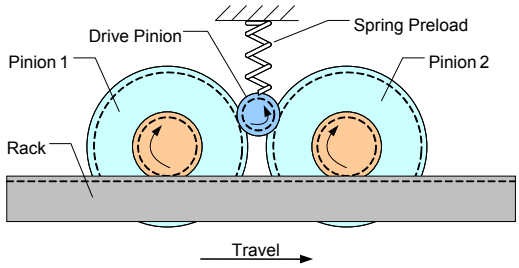
- Backlash is present in single pinion systems
- For low forces, a split preloaded pinion can be used
  - The internal tube acts as a torsional spring



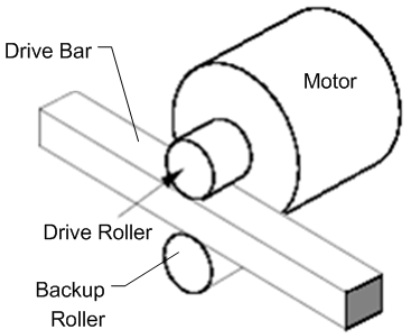
Source: Alexander Slocum, *Precision Machine Design*

Rack and Pinion Drives (contd.)

- For high force systems, a dual pinion system can be used:
  - Input shaft as a beam spring to drive two rollers:
    - Flexural force from beam spring causes input pinions to counter-rotate
    - Input pinion and motor can be mounted on cantilever beam that acts as a spring loaded pivot arm
    - Input torque causes input pinions to drive the main gear



Friction Drives



Source: Alexander Slocum, *Precision Machine Design*

- A wheel (capstan) driving a flat bar supported by a back up roller or hydrostatic flat pad bearing

Friction Drives (contd.)

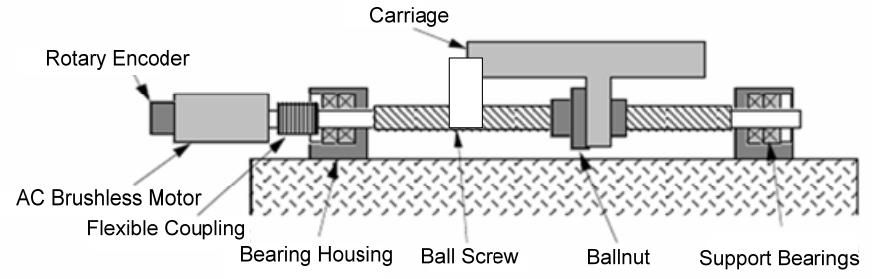
- Ideally, use hydrostatic bearings to support the drive roller shaft, and a hydrostatic flat pad bearing
- Accurate rollers are required to maintain a constant preload, transmission ratio, and constant torque
- A properly designed and manufactured friction drive can achieve nanometer resolution of motion
  - More common before linear electric motors were well developed
  - Still useful for long range of motion systems
- When a direct friction drive is properly aligned:
  - Only a pure radial bearing is needed to support the motor
  - Axial motion (walking) of the axially unrestrained shaft is an indicator of misalignment

Friction Drives (contd.)

- Friction drives' desirable properties include:
  - Minimal backlash and dead band (due to elastic deformation)
  - Low drive friction
  - Uncomplicated design
- Their undesirable properties include:
  - Low drive force capability
  - Low to moderate stiffness and damping
  - Minimal transmission gain (low motor speed makes them more susceptible to torque ripple)
  - High sensitivity to drive bar cleanliness
  - Frictional polymers can form on dry-running systems
  - As the capstan rolls, it compresses organic molecules in the air onto the drive bar which builds up a layer
    - This layer is not uniform and causes a bumpy ride and velocity control problems
  - Running the system with a tractive lubricant (e.g., Monsanto's Santotrac™):
    - Increases coefficient of friction
    - Prevents frictional polymer buildup

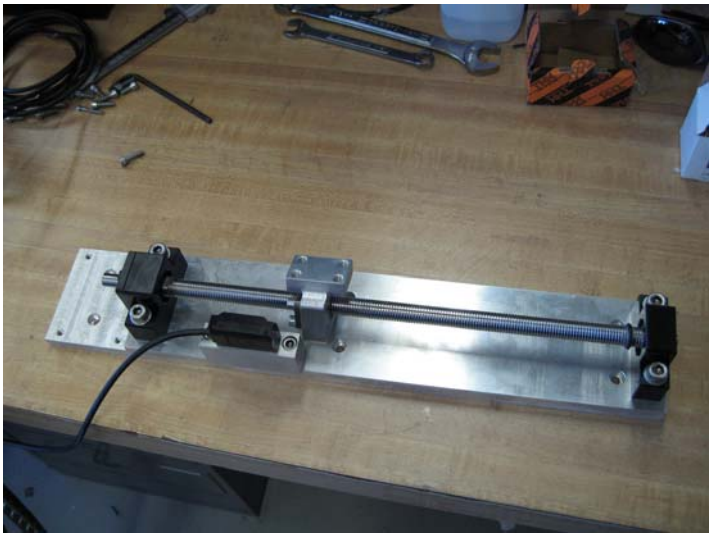
Lead Screws

- Lead screw principle has been used for centuries to convert rotary motion into linear motion with a high transmission ratio
- Modern lead screw driven servo system:



Source: Alexander Slocum, *Precision Machine Design*

Modular Ball Screw Drive System



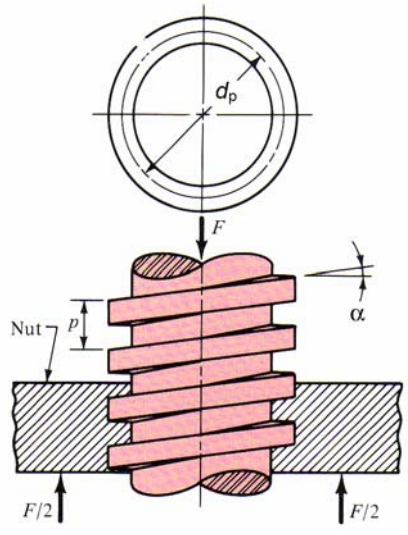
Source: Precision Design Lab

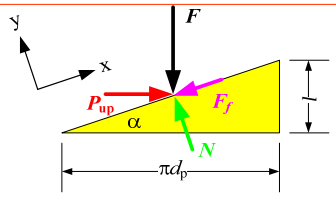
Lead Screws (contd.)

- There are many types of lead screws that are available including:
  - **Sliding contact thread lead screws**
  - Traction drive lead screws
  - Oscillatory motion lead screws
  - Non-recirculating rolling element lead screws
  - **Ball screws**
  - Planetary roller lead screws
  - Wallowing thread screws
  - Hydrostatic lead screws

The Mechanics of Power Screws - Lifting

Lifting load F





Summing up forces in x and y:

$$\sum F_x = P_{up}\cos\alpha - F\sin\alpha - F_f = 0 \quad (1)$$

$$\sum F_y = -F\cos\alpha + N - P_{up}\sin\alpha = 0 \quad (2)$$

The frictional force is found as:

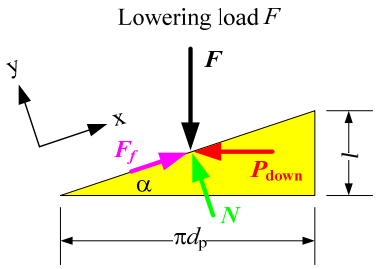
$$F_f = \mu N \quad (3)$$

(3) in (2) and in (1) and solve for P_{up}:

$$P_{up} = \frac{F(\sin\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\sin\alpha} \quad (4)$$

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

The Mechanics of Power Screws - Lowering



Summing up forces in *x* and *y*:

$$\sum F_x = -P_{down}\cos\alpha - F\sin\alpha + F_f = 0 \quad (5)$$

$$\sum F_y = -F\cos\alpha + N + P_{down}\sin\alpha = 0 \quad (6)$$

(5) in (6) and solve for P_{down}:

$$P_{down} = \frac{F\left(\mu\cos\alpha - \sin\alpha\right)}{\cos\alpha + \mu\sin\alpha} \quad (7)$$

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Lead Screws – Torque Calculations

Using: $$\tan\alpha = \frac{l}{\pi d_p} \quad (8)$$

(8) in (4):

$$P_{down} = \frac{F\left(\mu - \dfrac{l}{\pi d_p}\right)}{1 + \dfrac{\mu l}{\pi d_p}} \quad (9)$$

(8) in (7):

$$P_{up} = \frac{F\left(\dfrac{l}{\pi d_p} + \mu\right)}{1 - \dfrac{\mu l}{\pi d_p}} \quad (10)$$

The torque *T* applied can be written as the product of force *F* and pitch diameter d_p

$$T = \frac{F d_p}{2} \quad (11)$$

(11) in (9):

$$T_{up} = \frac{F d_p}{2}\left(\frac{l + \pi\mu d_p}{\pi d_p - \mu l}\right) \quad (12)$$

(11) in (10):

$$T_{down} = \frac{F d_p}{2}\left(\frac{\pi\mu d_p - l}{\pi d_p + \mu l}\right) \quad (13)$$

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Lead Screws – Torque Calculations (contd.)

For the case of an **Acme** screw, the wedging effect of the thread angle needs to be considered. This is done by dividing the frictional terms in Eq. (12) and (13) by *cos*θ.

$$T_{up} = \frac{Fd_p}{2}\left(\frac{l\cos\theta + \pi\mu d_p}{\pi d_p \cos\theta - \mu l}\right) \quad (14) \qquad T_{down} = \frac{Fd_p}{2}\left(\frac{\pi\mu d_p - l\cos\theta}{\pi d_p \cos\theta + \mu l}\right) \quad (15)$$

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Self Locking

Screws are said to be self-locking if the friction inside the nut is large enough to create a positive torque. Self-locking resists spinning of the nut when an external force *F* is applied to the threaded shaft.

Thus the condition for self-locking is:

$$\pi\mu d_p > l\cos\theta \quad (19)$$

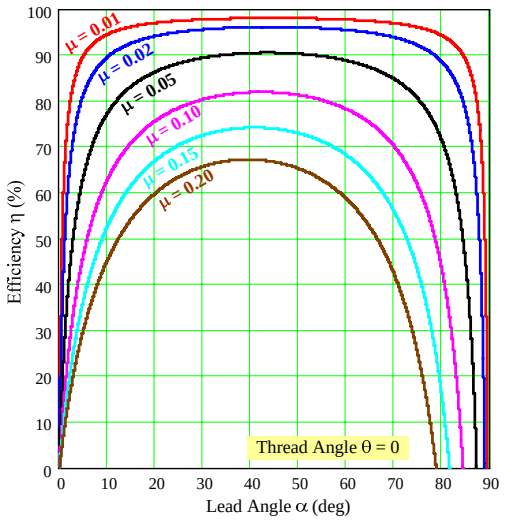
Lead Screws (contd.)

- The efficiency η is:

$$\eta = \frac{\cos\theta - \mu\tan\alpha}{\cos\theta + \mu\cot\alpha} \qquad (17)$$

For square thread:

$$\eta = \frac{1 - \mu\tan\alpha}{1 + \mu\cot\alpha} \qquad (18)$$



Source: Juvinall RC, Marshek KM, *Fundamentals of Machine Component Design*

Stresses in Lead Screws

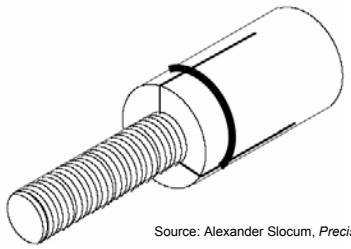
- The force that is generated creates tensile and torsional shear stresses
- The thread root is a stress concentration area (a factor of 2-3), which is somewhat mitigated when the threads are rolled (as opposed to cut)
- Assuming a thread root diameter r_{tr} , the stresses are:

$$\sigma_{tensile} = \frac{F_{tensile}}{\pi r_{tr}^2} \qquad \tau_{shear} = \frac{2T}{\pi r_{tr}^3}$$

The equivalent (Von Mises) stress:

$$\sigma_{equivalent} = \sqrt{\sigma_{tensile}^2 + 3\tau_{shear}^2}$$

Sliding Contact Lead Screws



Source: Alexander Slocum, *Precision Machine Design*

- Molded plastic nuts are often split and preloaded by an O-ring which puts circumferential pressure on the nut
- Commercially available thread ground and lapped sliding contact thread lead screw assemblies may have nuts preloaded against each other
- They may have split nuts that are preloaded with a circumferential spring

Sliding Contact Lead Screws (contd.)

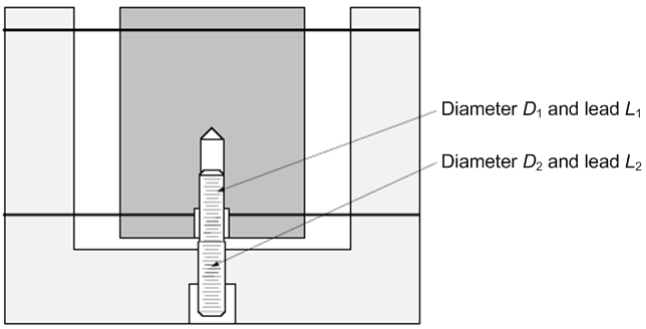
- Range from least expensive (machine finished) to most expensive (lapped) lead screws.
- Usually the nut is made of a bearing brass or bronze but can also be made from PTFE or the nut can be replicated.
- For low force applications, the nut can be bored without threads and then have axial slits cut into it.
  - The nut is then placed over a fine pitch lead screw (e.g. 100 threads per inch).
  - O-rings circumferentially clamp the nut and the screw will then make its own impressions into the nut.

Sliding Contact Lead Screws (contd.)

- The coefficient of friction between a sliding thread contact lead screw can range from:
  - 0.1 for a greased nut
  - 0.05 for a lightly loaded lapped thread
- Load capacity is an order of magnitude less than for a ball screw
- However, the lapped continuous contact thread provides much greater smoothness of motion
- Allows a ground and lapped screw to achieve sub-micro-inch motion once initial stick slip has been overcome
- Sliding contact threads can also be replicated in-place, and sometimes can even be used to make a hybrid screw

Differential Lead Screws

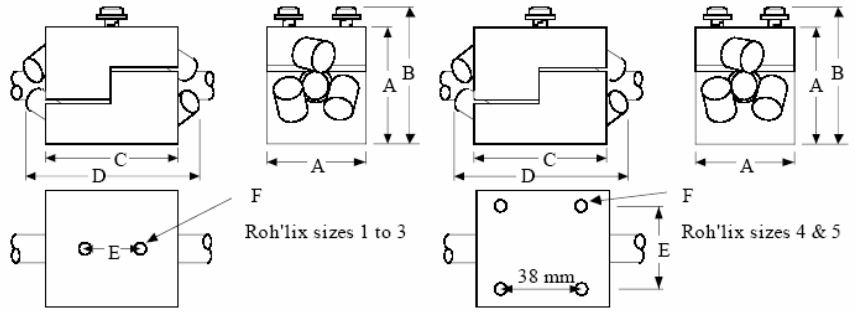
- For very large transmission ratio if lead L_1 and lead L_2 have same hand
- For very small transmission ratio if lead L_1 and lead L_2 have opposite hand



Source: Alexander Slocum, *Precision Machine Design*

Traction Drive Lead Screw

- Cam roller-type traction drive lead screw (Courtesy of Zero-Max a unit of Barry Wright):



Source: Alexander Slocum, *Precision Machine Design*

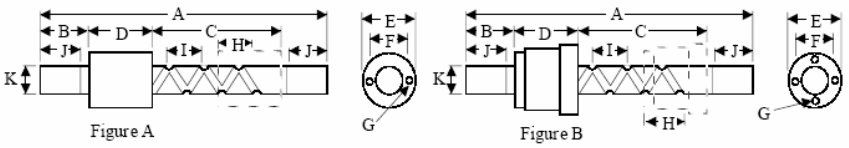
- The cam rollers' axes are inclined to the axis of the shaft
  - The angle of inclination determines the lead

Traction Drive Lead Screw (contd.)

- The efficiency of this type of screw is generally on the order of 0.9 (90%) and load capacity is low
- If overloaded, the nut will slip
  - For a CMM that uses linear encoders for accuracy, the slip capability is ideal from a crash/safety perspective
- For applications requiring moderate accuracy and load capability with high efficiency and low cost
  - The shaft is smooth and round so it is exceptionally easy to seal
- Beware of the buildup of frictional polymers on the shaft!

Oscillatory Motion Lead Screws

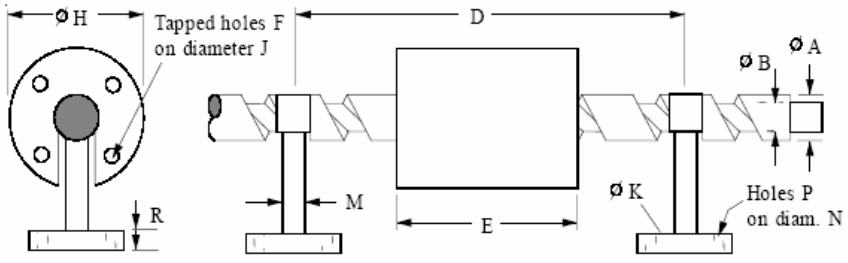
- For applications requiring linear oscillatory motion over a fixed path
- Various turnaround curve profiles are available which allows the dwell to be chosen for the application
- A typical application would be in photocopying machines
- Characteristics of a lead screw that provides oscillating motion (Courtesy of Norco Inc.):



Source: Alexander Slocum, *Precision Machine Design*

Non-Recirculating Rolling Element Lead Screws

- Metric Rollnuts for very long range of motion actuation (Courtesy of Norco Inc.):



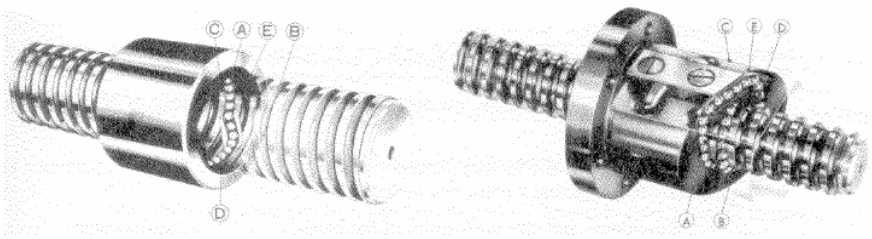
Source: Alexander Slocum, *Precision Machine Design*

Non-Recirculating Rolling Element Lead Screws (contd.)

- Primary feature of the Rollnut is the rolling elements are fixed and can pass over discontinuities in the shaft
- A very long shaft can be spliced together and suspended by shaft hangers
- The non-rotating nut can travel tens of meters without worry of shaft deflection and critical speeds except for regions between hangers
- Very useful in material handling systems

Ball Screws

- There are two main types of ball screw nuts (Courtesy of NSK Corp.):



- The tube-type uses a pick-up finger to gather the balls as they exit the nut's thread helix and a tube to direct them back to the beginning of the thread helix.
  - Large lead/diameter ratios are possible (2:1)
  - The speed of the shaft (rpm) x shaft diameter (mm) is limited to about 90,000

Source: Alexander Slocum, *Precision Machine Design*

Ball Screws (contd.)

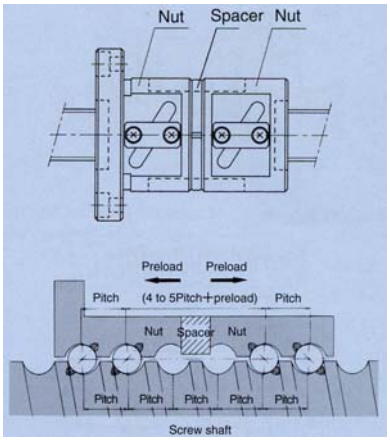
- Ball screws are perhaps the most common type used in industrial machinery and precision machines
- Ball screws can be used to easily achieve repeatability on the order of one micron
- Specially manufactured and tested ball screws can attain submicron motion resolution
- Very high efficiency is obtained by using rolling steel balls to transfer loads from the screw shaft to the nut threads
  - Smaller spacer balls in-between load-carrying balls increases the allowable speed and the smoothness of operation (greater resolution)
  - The use of spacer balls halves the number of load carrying balls, so load capacity is cut by roughly 50%

Preloading Methods

- Preloading removes backlash in ball screw nuts
- There are four basic ways:
  - Tensile preloading is created by inserting an oversize spacer between two nuts and then clamping the nuts together
  - Compressive preloading uses an undersize spacer between two nuts that are clamped together
  - P-type preloading uses oversize balls, and a single nut to reduce cost
  - Z-type preloading is also obtained with a single nut by shifting the lead between ball circuits
- Preloading reduces the load capacity, increases the nut stiffness, and increases internal friction (heat!!!)

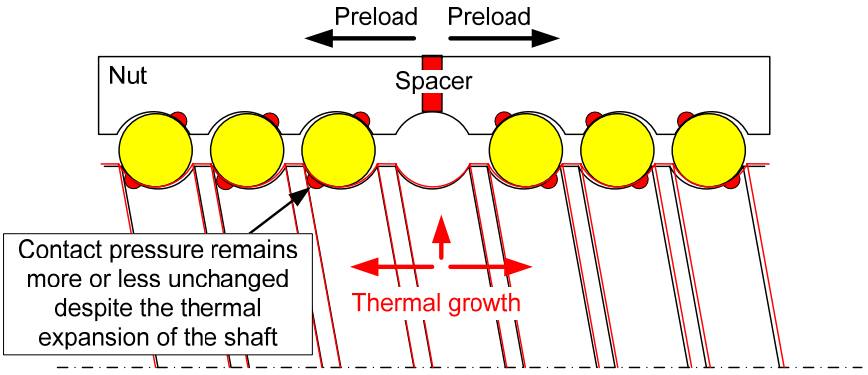
Tensile Preloading

- Tensile preloading is created by inserting an oversize spacer between two nuts and then clamping the nuts together:
  - One nut takes loads in one direction and the other takes loads in the other direction
  - This creates a back-to-back mounting effect that is thermally stable for a rotating shaft design
  - Just like for rotary ball bearings, a back-to-back nut is more sensitive to misalignment errors



Source: THK Co., Ltd.

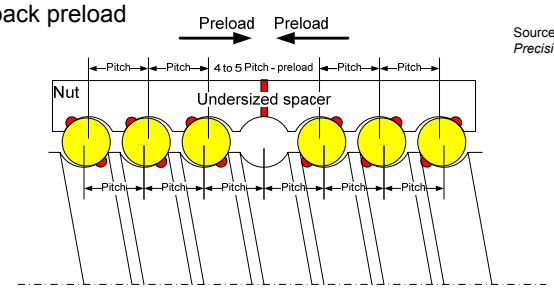
Tensile Preloading – Thermal Stability



Scenario: Temperature of the shaft is greater than that of the nut. Because the contact pressure remains largely unchanged despite the thermal expansion of the shaft, the system is called *thermally stable*.

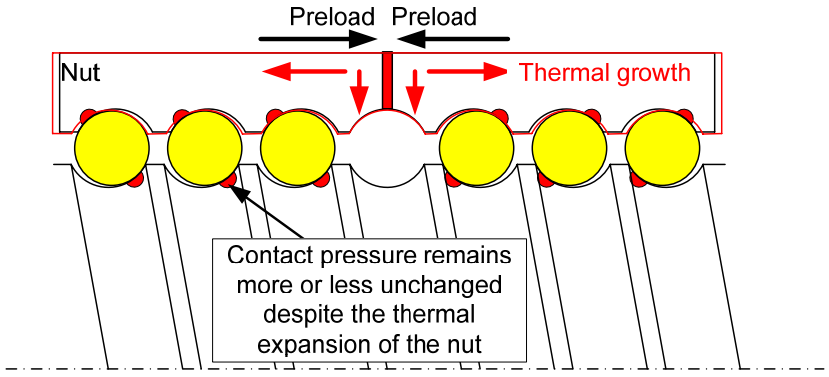
Compressive Preloading

- Compressive preloading uses an undersize spacer between two nuts.
  - This creates a face-to-face mounting situation:
    - Thermally stable if the nut is likely to be hotter than the lead screw
  - If the nut has an angular displacement imposed on it due to mounting errors:
    - The ball loads will be lower and life will be longer than with a back-to-back preload



Source: Alexander Slocum, *Precision Machine Design*

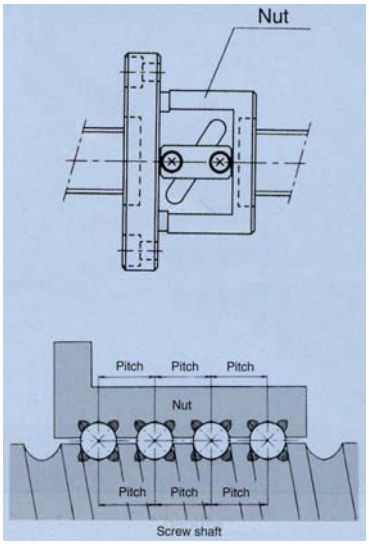
Compressive Preloading – Thermal Stability



Scenario: Temperature of the nut is greater than that of the shaft. Because the contact pressure remains largely unchanged despite the thermal expansion of the nut, the system is called *thermally stable*.

P-Type Preloading

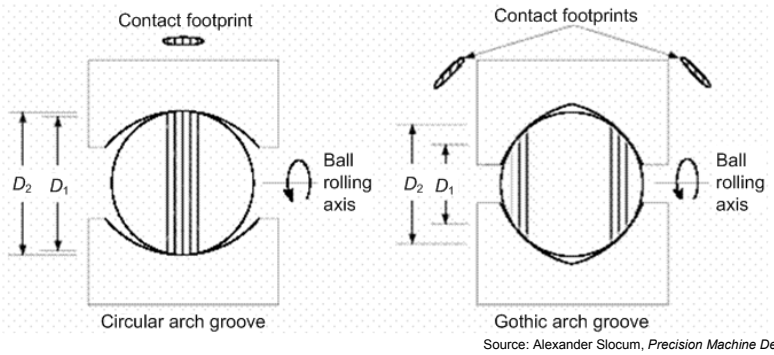
- P-type preloading uses oversize balls, and a single nut to reduce cost
- Preload is obtained by four point contact between the balls and the Gothic arch thread shape of the shaft and the nut
  - This greatly increases the amount of skidding (differential spin) the ball is subjected to
  - Because of skidding, this type of preload should not be used for precision systems
    - If an axis is primarily loaded in one direction (gravity), the P-type preloading is effective and economical



Source: THK Co., Ltd.

P-Type Preloading (contd.)

- A circular arch groove typically has 3% slip during rolling compared to 40% for a Gothic arch groove:

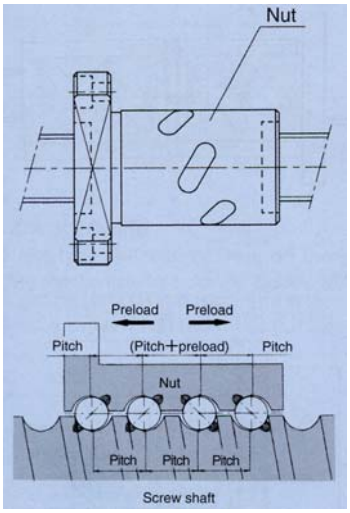


Source: Alexander Slocum, *Precision Machine Design*

- As sensitive to misalignment as is tensile preloading

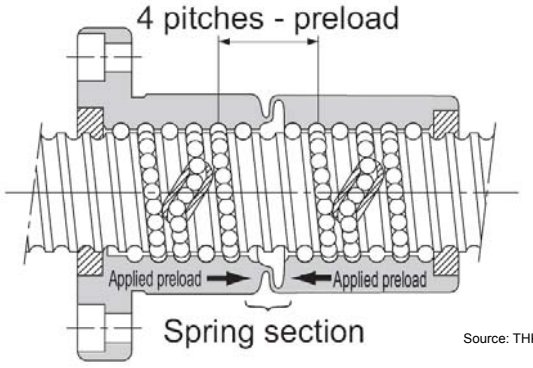
Z-Type Preloading

- Z-type preloading is also obtained with a single nut by shifting the lead between ball circuits
  - This creates two point contact between the balls and the grooves
  - Tensile or compressive mode
  - Two types:
    - Two circuits with spaced lead
    - Single circuit with the circuit having a midway transition point or "skip" by the preload amount
  - No differential spin occurs, hence high degree of accuracy for an extended period of time



Source: THK Co., Ltd.

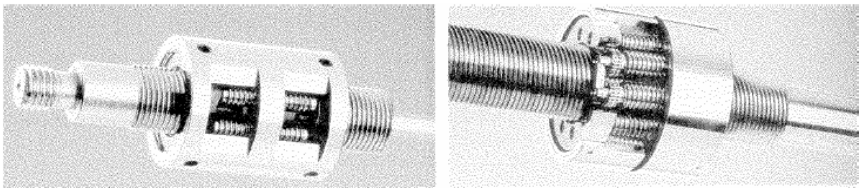
Spring Type Preload



Source: THK Co., Ltd.

- Simple preload method
- Thermally stable
- Useful if stiffness is not required

Roller Screws



Source: Alexander Slocum, *Precision Machine Design*

- Use rollers instead of balls to convert rotary to linear motion and transfer forces between the nut and the shaft
- Rollers have a single thread with a pitch equal to the lead screw's apparent pitch (real pitch / number of leads)
- The rollers mesh with both the screw and the nut

Recirculating Roller Screws

- The nut has a region that allows the rollers to move radially out of the shaft thread
- A cam sends the rollers back to start a new circuit
- Allows for very fine pitch threads (2 mm)

Some Facts about Roller Screws

- Roller screw size ranges:
  - Diameter ranging from 3.5 to 200 mm
  - Pitch ranging from 1 to 40 mm
  - Single nut dynamic load capacity ranging from 5300 N to 753 kN
- The nut can be made in one piece without preload, or in two pieces if preload is required
- Maximum speed of a planetary roller lead screw is ω (rpm) x D (mm) = 140,000 (twice that of a ball screw)
- The large number of contacting threads creates a large averaging effect
- This also gives roller screws load and stiffness capabilities many times higher than similar diameter ball screws

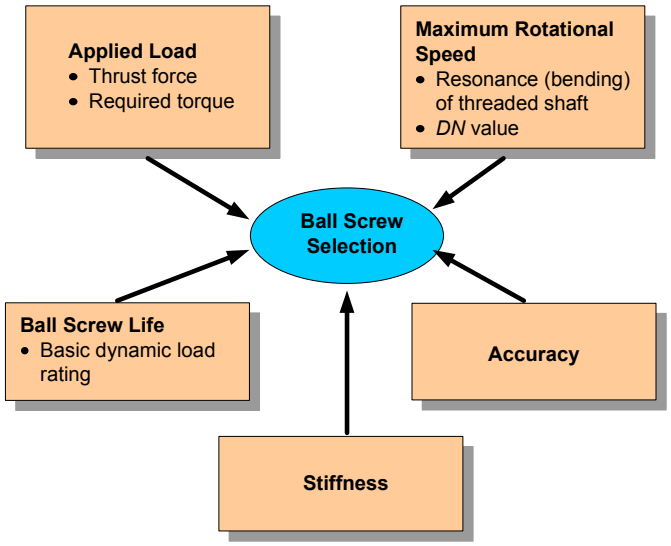
Some Facts about Roller Screws (contd.)

- Depending on the screw, a roller screw may cost one to three times as much as a ball screw
  - Hertz contact area is 3x that of a same size ball screw
  - Load capacity is 3x that of a same size ball screw
  - Life = $(F/Fmax)^3$, so life is 27x that of a same size ball screw
- Roller screws are manufactured with accuracies and efficiencies equal to those of ball screws
  - CAREFUL: Tens-of-hours run-in may be required to ensure the very stiff nuts are properly preloaded!
- Planetary roller screws are much more quiet than ball screws

Ball Screw Selection and Calculations

ME EN 7960 – Precision Machine Design
Topic 4

Ball Screw Selection Criteria



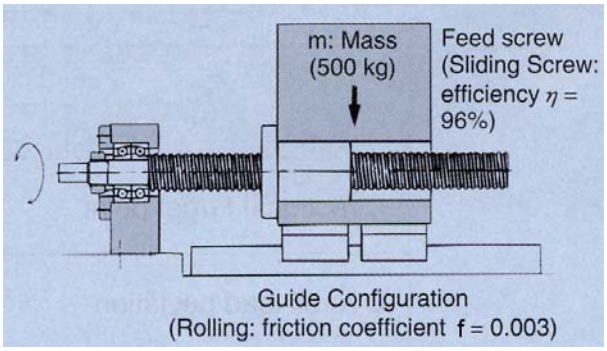
Based on Load

- A ball screw transforms rotational motion into translational motion. As a result, the shaft is subject to loads:
  - Thrust force (the sum of all external forces such as machining load, gravity, friction, inertial forces, etc.).
  - Torque required to generate the thrust force.

Driving Torque to Obtain Thrust

$$T = \frac{F_a l}{2\pi\eta}$$

T: driving torque [Nm]
F_a: thrust force [N]
l: screw lead [m]
η: efficiency



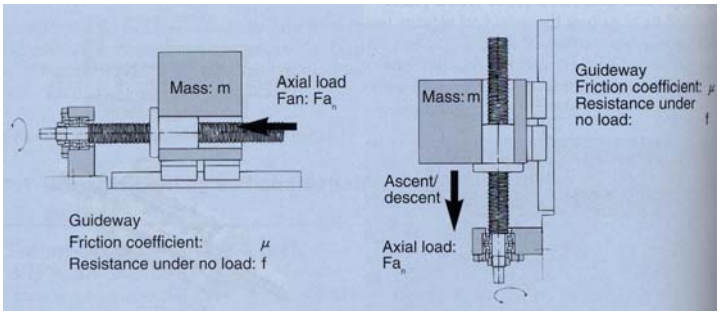
Source: THK Co., Ltd.

Required Thrust

- The thrust is the sum of all forces acting in the axial direction.

$$F_a = F_M + F_f + F_i + F_g$$

F_M: Machining force [N]
F_f: Frictional force [N]
F_i: Inertial force [N]
F_g: Gravitational force [N]



Source: THK Co., Ltd.

Stresses from Applied Loads

$$\sigma_{axial} = \frac{F_a}{\pi r_{tr}^2} \qquad \tau_{torsional} = \frac{2T}{\pi r_{tr}^3}$$

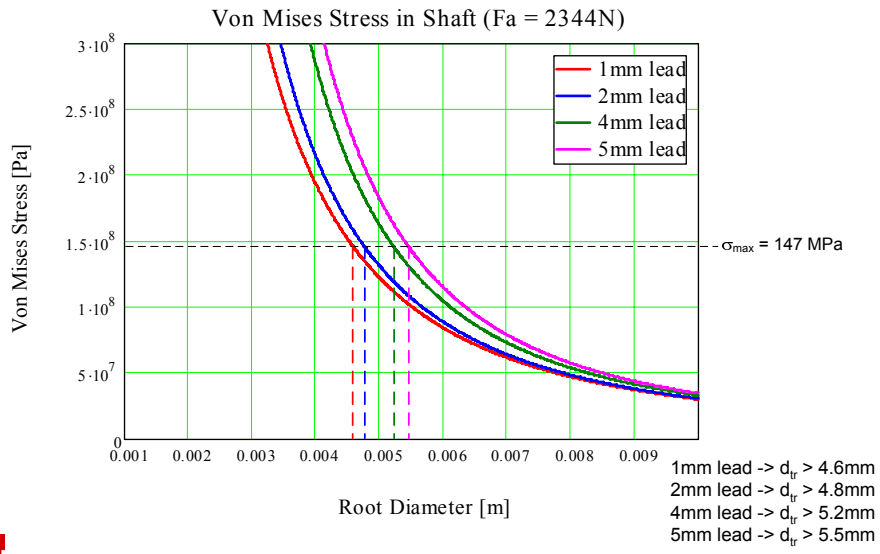
The equivalent (Von Mises) stress:

$$\sigma_{eq} = \sqrt{\sigma_{axial}^2 + 3\tau_{torsional}^2}$$

$$\rightarrow \sigma_{eq} = \frac{4F_a}{\pi d_{tr}^2}\sqrt{1 + \frac{12l^2}{\pi^2 d_{tr}^2 \eta^2}}$$

σ_{max}: Permissible stress [147 MPa]

Graphic Solution



1mm lead -> d_{tr} > 4.6mm
2mm lead -> d_{tr} > 4.8mm
4mm lead -> d_{tr} > 5.2mm
5mm lead -> d_{tr} > 5.5mm

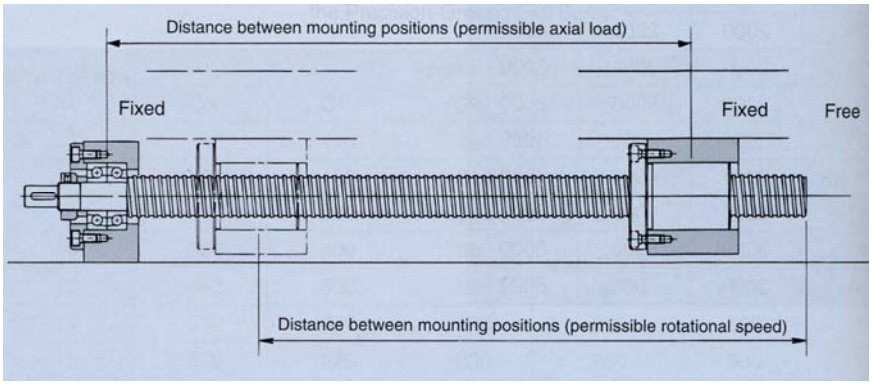
Permissible Compressive Load

- Buckling Load

$$P_1 = \frac{\lambda \pi^2 EI}{l_b^2}$$

P_1:	Buckling load [N]	
l_b:	Distance between mounting positions [m]	
E:	Elastic modulus [Pa]	
I:	Second moment of inertia [m^4]	
λ:	Support factor	
	Fixed – free:	**λ = 0.25**
	Fixed – supported:	**λ = 2.0**
	Fixed – fixed:	λ = 4.0

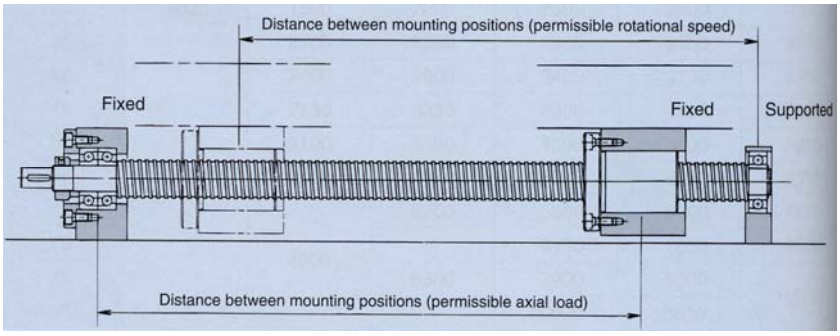
Fixed-Free Mount



Source: THK Co., Ltd.

Inexpensive but only applicable for short ball screws and/or slow speeds.

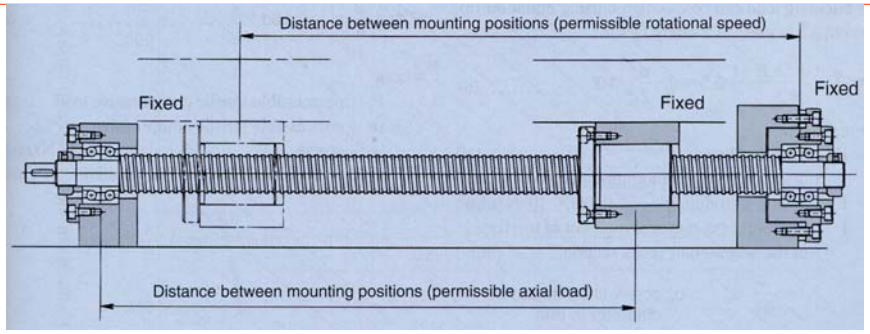
Fixed-Supported Mount



Source: THK Co., Ltd.

Most commonly used mounting setup.

Fixed-Fixed Mount



Source: THK Co., Ltd.

Overconstrained mounting setup for applications where high stiffness, accuracy, and high shaft speed is required. Ball screw needs to be pre-stretched to avoid buckling in the case of thermal expansion.

Basic Static Load Rating C_{oa}

- When ball screws are subjected to excessive loads in static condition (non rotating shaft), local permanent deformations are caused between the track surface and the steel balls.
- When the amount of this permanent deformation exceeds a certain degree, smooth movement will be impaired.

$$C_{oa} \geq f_s F_a$$

C_{oa}: Basic static load rating [N, kgf, lbf]
f_s: Static safety factor
F_a: Load on shaft in axial direction [N, kgf, lbf]

Use conditions	**f_s (lower limit)**
Normal operation	1.0 – 2.0
Operation with impacts and vibrations	2.0 – 3.0

Permissible Speed

- When the speed of a ball screw increases, the ball screw will approach its natural frequency, causing a resonance and the operation will become impossible.

$$n_c = \frac{60\lambda^2}{2\pi l_b^{\,2}}\sqrt{\frac{EI}{\rho A}} = \frac{15\lambda^2 d_{tr}}{2\pi l_b^2}\sqrt{\frac{E}{\rho}}$$

n_c:	Critical speed [min^{-1}]	
l_b:	Distance between supports [m]	
E:	Elastic modulus [Pa]	
I:	Second moment of inertia [m^4]	
ρ:	Density [kg/m^3]	
A:	Root cross sectional area [m^2]	
λ:	Support factor	
	Fixed – free:	**$\lambda = 1.875$**
	Supported – supported:	$\lambda = 3.142$
	Fixed – supported:	**$\lambda = 3.927$**
	Fixed – fixed:	$\lambda = 4.730$

Spindle Speed and *DN* Value

- Shaft speed

$$n = \frac{v_a}{l}$$

n:	Revolutions per second [s^{-1}]
v_a:	axial speed [m/s]
l:	lead [m]

- DN Value. Unless specified otherwise:

$$DN \le 70000$$

D:	Ball circle diameter [mm]
N:	Revolutions per minute [min^{-1}]

Dynamic Load Rating C_a and *Life*

- The basic load rating C_a is the load in the shaft direction with 90% of a group of the same ball screws operating individually will reach a life of 10^6 (1 million) revolutions.

$$L = \left(\frac{C_a}{f_w F_a} \right)^3 \times 10^6$$

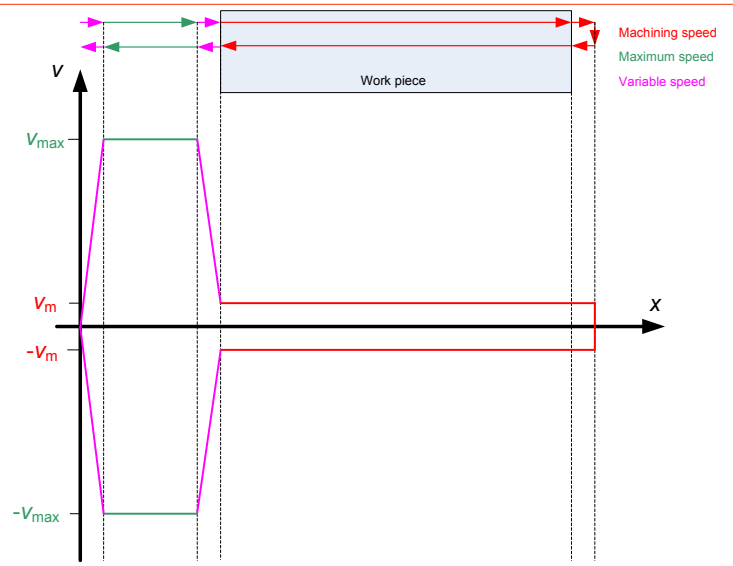
L:	Rotation life [rev]
C_a:	Basic dynamic load rating [N, kgf, lbf]
f_w:	Load factor
F_a:	Load in shaft direction [N, kgf, lbf]

Use conditions	f_w
Smooth movement without impacts	1.0 – 1.2
Normal movements	1.2 – 1.5
Movement with impacts and vibrations	1.5 – 2.5

Example

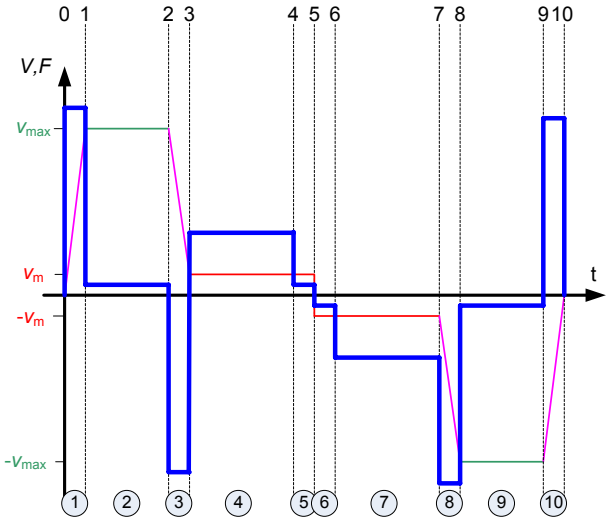
- Mass of axis: 350kg
- Maximum velocity: 20m/min
- Acceleration time: 0.05s
- Bearing friction factor: 0.003
- Machining force: 500N
- Length of work piece: 500mm
- Length of travel at maximum speed: 100mm
- Orientation of axis: horizontal

Machining Profile of Making a Slot



$v_0 = 0$
$v_1 = v_{max}$
$v_2 = v_{max}$
$v_3 = v_m$
$v_4 = v_m$
$v_5 = 0$
$v_6 = -v_m$
$v_7 = -v_m$
$v_8 = -v_{max}$
$v_r = -v_{max}$
$v_{10} = 0$

Load Profile for Making a Slot



$v_0 = 0$
$v_1 = v_{max}$
$v_2 = v_{max}$
$v_3 = v_m$
$v_4 = v_m$
$v_5 = 0$
$v_6 = -v_m$
$v_7 = -v_m$
$v_8 = -v_{max}$
$v_r = -v_{max}$
$v_{10} = 0$

$F_{a1} = F_i + F_f = 2344N$
$F_{a2} = F_f = 10N$
$F_{a3} = -F_i + F_{fi} = -2324N$
$F_{a4} = F_m + F_f = 510N$
$F_{a5} = F_f = 10N$
$F_{a6} = -F_f = -10N$
$F_{a7} = -F_m - F_f = -510N$
$F_{a8} = -F_i - F_f = -2344N$
$F_{a9} = -F_f = -10N$
$F_{a10} = F_i - F_f = 2324N$

Running Lengths Depending on Usage

Running distance during acceleration: $l_{accl} = \frac{(v_1 + v_2)t_{accl}}{2}$

Running distance during deceleration: $l_{decl} = \frac{(v_1 + v_2)t_{decl}}{2}$

Mean Axial Force

Determine mean axial load in the positive direction by collecting all individual, positive axial loads.

$$F_{a,mean+} = \sqrt[3]{\frac{\sum_{i+} F_{ai+}^3 l_{i+}}{\sum_i l_i}}$$

Determine mean axial load in the negative direction by collecting all individual, negative axial loads.

$$F_{a,mean-} = \sqrt[3]{\frac{\sum_{i-} \left|F_{ai-}^3\right| l_{i-}}{\sum_i l_i}}$$

Determine mean axial load: $F_{a,mean} = \frac{F_{a,mean+} + F_{a,mean-}}{2}$

Load Profile Based on Utilization

Mean axial force:

$$F_{a,mean} = \sqrt[3]{\frac{F_m^3 l_m + F_{uni}^3 l_{uni} + F_{accl}^3 l_{accl}}{l_b}}$$

F_m:	Machining force
F_{uni}:	Force at constant velocity (not machining)
F_{accl}:	Maximum force during acceleration and deceleration
l_m:	Total travel per cycle during machining
l_{uni}:	Total travel per cycle at constant velocity
l_{accl}:	Total travel per cycle during acceleration and deceleration
l_b:	Length of ball screw

Total travel length:

$$l_b = l_m + l_{uni} + l_{accl}$$

Travel length:

$$l_m = q_m l_b$$

$$l_{uni} = q_{uni} l_b$$

$$l_{accl} = q_{accl} l_b$$

Load Profile Based on Utilization (contd.)

Utilization:

$$q_m + q_{uni} + q_{accl} = 1$$

q_m:	Percentage per cycle spent machining (typically 0.5 – 0.9)
q_{uni}:	Percentage per cycle spent at constant velocity (typically 0.05 – 0.45)
q_{accl}:	Percentage per cycle spent during acceleration and deceleration (typically 0.05 – 0.1)

Dynamic Load Rating C_a and *Life*

- When the rotation life L has been obtained, the life time can be obtained according to the following formula if the stroke length and the operation frequency are constant:

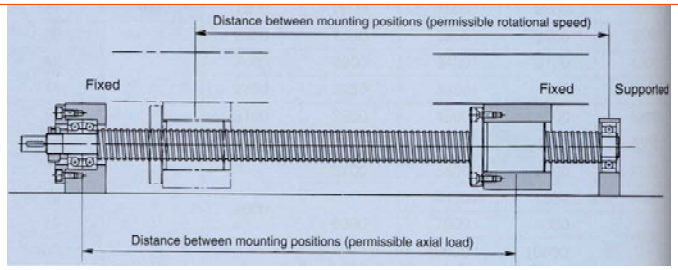
$$L_h = \frac{L}{60 n_m}$$

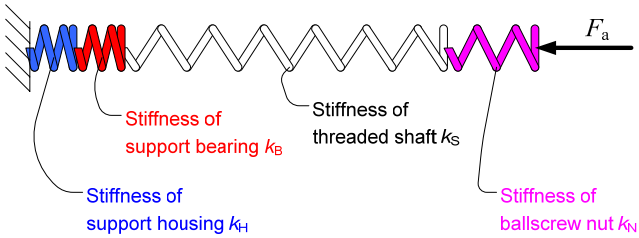
L: Rotation life [rev]
L_h: Life time [hr]
n_m: mean rotational speed [min^{-1}]

$$n_m = \frac{\sum_i (n_i l_i)}{\sum_i l_i}$$

n_i: rotational speed at phase i [min^{-1}]
l_i: distance traveled at phase i [m]

Axial Rigidity





Axial Rigidity

Fixed-free
Fixed-supported

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_N} + \frac{1}{k_B} + \frac{1}{k_H}$$

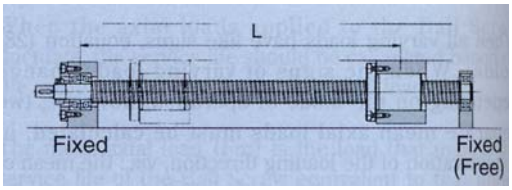
Fixed-fixed

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_N} + \frac{1}{k_{B1} + k_{B2}} + \frac{1}{k_{H1} + k_{H2}}$$

k:	Axial rigidity of linear motion system [N/m]
k_s:	Axial rigidity of screw shaft [N/m]
k_N:	Axial rigidity of nut [N/m]
k_B:	Axial rigidity of support bearing [N/m]
k_H:	Axial rigidity of support housing [N/m]

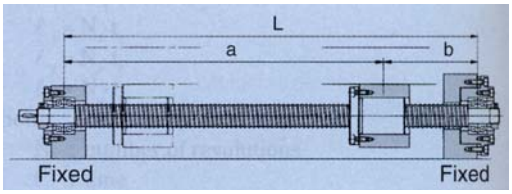
Ball Screw Selection Procedure

- Axial rigidity of shaft



Fixed-free and fixed-supported:

$$k_s = \frac{AE}{L}$$



Fixed-fixed:

$$k_s = \frac{AEL}{ab}$$

$$k_{s,\min} = \frac{4AE}{L}$$

Ball Screw Accuracy

- Applicable if used in combination with rotary encoders.

Table 1 Accumulated reference lead error and variation (tolerance) Unit: μm

Accuracy grade		Ground products C3		Ground products C5		C7	Rolled products C8	Rolled products C10
Thread part length (mm)		Accumulated reference lead error	Variation	Accumulated reference lead error	Variation	Accumulated reference lead error	Accumulated reference lead error	Accumulated reference lead error
Over	Up to							
—	315	12	8	23	18	±0.05mm/ 300mm (No other specifications)	±0.1mm/ 300mm (No other specifications)	±0.21mm/ 300mm (No other specifications)
315	400	13	10	25	20			
400	500	15	10	27	20			
500	630	16	12	30	23			
630	800	18	13	35	25			
800	1000	21	15	40	27			
1000	1250	24	16	46	30			
1250	1600	29	18	54	35			
1600	2000	35	21	65	40			
2000	2500	41	24	77	46			
2500	3150	50	29	93	54			
3150	4000	—	—	115	65			
4000	5000	—	—	140	77			

Source: THK Co., Ltd.

Overall Error

- The overall error is the summation of a number of individual errors:
  - Non-uniformity of threaded shaft.
  - Resolution/accuracy of rotary encoder.
  - Axial compliance of ball screw assembly.
  - Torsional compliance of threaded shaft.

$$\delta_a = \delta_S + \frac{l}{N_{rot}} + \frac{F_a}{k_{axial}} + \frac{32 T l_b}{\pi d_{tr}^4 G} \cdot \frac{l}{2\pi}$$

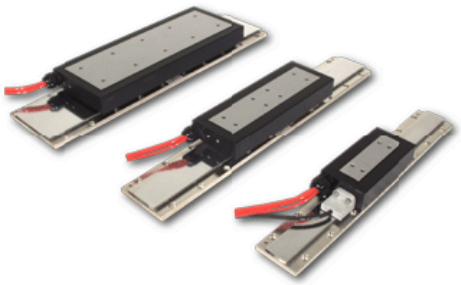
δ_a:	overall linear error [m]	k_{axial}:	overall linear stiffness [N/m]
δ_s:	ball screw non-linearity [m]	T:	applied torque [Nm]
l:	lead [m]	l_b:	ball screw length [m]
N_{rot}:	resolution of rotary encoder [pulses/rev]	d_{tr}:	root diameter of shaft [m]
		G:	shear modulus [Pa]
F_a:	thrust force onto system [N]		

Alternative Linear Motion Systems

ME EN 7960 – Precision Machine Design
Topic 5

Iron Core Linear Motors

- Provide actuation forces from 30 N (7 lbf) continuous to over 11,000 N (2,500 lbf) peak
- Two-part construction features a 3-phase laminated coil and a single sided magnet track consisting of rare-earth magnets mounted on a rigid steel base plate
- These linear motors deliver excellent performance in rapid point-to-point positioning applications while virtually eliminating the efficiency-robbing effects of friction, inertia, backlash and wear associated with rotary-to-linear drive mechanisms



Source: Anorad

Iron Less Linear Motors

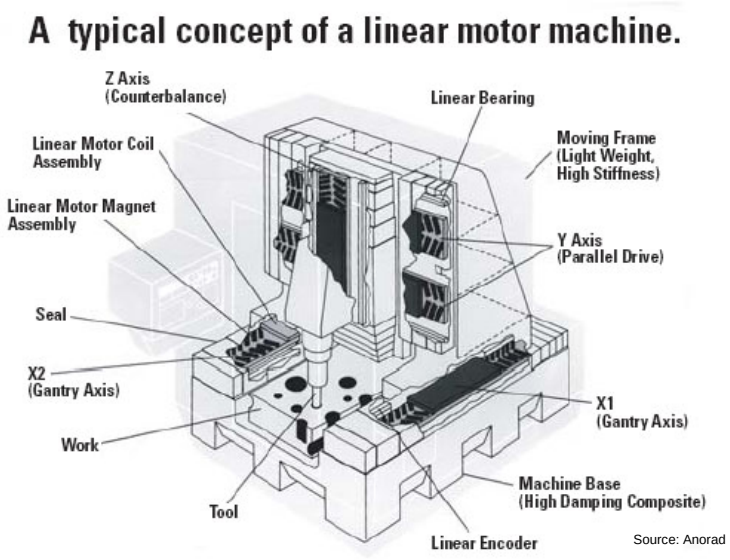
- Provide actuation forces from 8 N (2 lbf) continuous to nearly 2,000 N (450 lbf) peak
- Their two-piece assembly features a 3-phase ironless coil and a balanced dual-sided “U” shaped magnet channel
- Their ironless coil design eliminates cogging (magnetic détente) and makes them the optimal choice for extremely consistent velocity control in scanning-type applications and allows them to achieve the highest levels of positioning accuracy, repeatability and resolution

Source: Anorad

Linear Motors

- Pros
  - Direct drive minimizes the number of parts
  - Velocity and acceleration not limited by the actuator
    - Ranging from less than 1μm/s to more than 10m/s
  - Unlimited travel
  - Smoothness of motion
  - Accuracy and resolution is not limited by the actuator
  - Zero wear
  - Very simple installation
  - Very clean operation
- Cons
  - Optimal transmission ratio is not achieved
  - High currents, large thermal loads
  - Some motors create high attractive forces (10x the thrust force)
  - Vertical axes require safety breaks and/or counter balances
  - Much more expensive components ($10/lbf thrust vs. $2/lbf for a ball screw system).
  - Not self-locking, therefore needs constant servo control

Typical Application



Brushless DC Linear Motor

- Never a problem with an unused portion's brush surfaces corroding
- Good power density
- Non-contact generation of force
- Closed loop control provides resolution limited only by sensor and electronics
- Ironless-core design has flux crossing gap that is intersected by windings
  - No attractive force to load bearings
  - Requires a sandwiched design

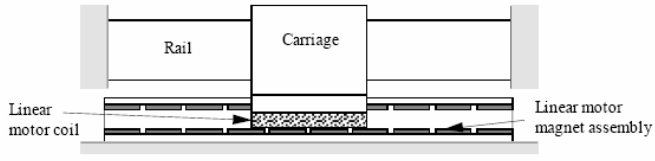
Brushless DC Linear Motor (contd.)

- The DC brush linear motor is ideal for long stroke, open or closed loop servo, linear motion applications with speeds from 1 in/s [25 mm/s] to 100 in/s [2.5 m/s]
- The motor is commutated using brushes on the moving permanent magnet secondary in conjunction with a stationary commutator bar on the coil assembly. This results in only the coils directly beneath the secondary with current flowing in them.
- The short moving brush assembly is magnetically attracted to the long stationary laminated coil assembly. A customer supplied bearing system is required to guide the moving secondary and to maintain a .025" [0.63mm] gap between the secondary and the coil assembly.

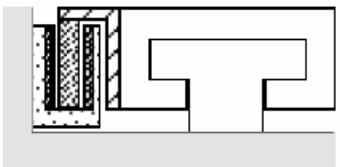
Source: H2W Technologies

Mountings

- Linear motor moving coil mounted to the underside of an air bearing carriage that rides on a simply supported rail:



- Linear motor moving coil mounted to the side of an air bearing carriage that rides on a T-shaped rail that is fully supported along its length

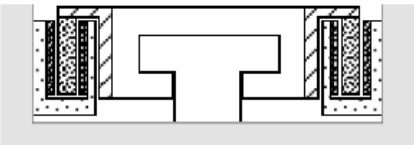


Source: Alexander Slocum, *Precision Machine Design*

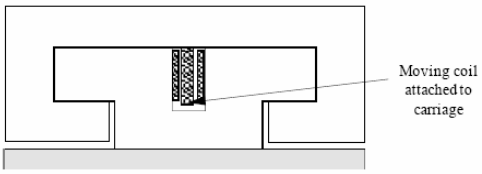
Warning: Motor force is not aligned with center of friction and center of mass. Moments will occur as a result, resulting in rotation of the slide.

Mountings (contd.)

- Linear motor moving coil mounted to the sides of an air bearing carriage that rides on a T-shaped rail that is fully supported along its length



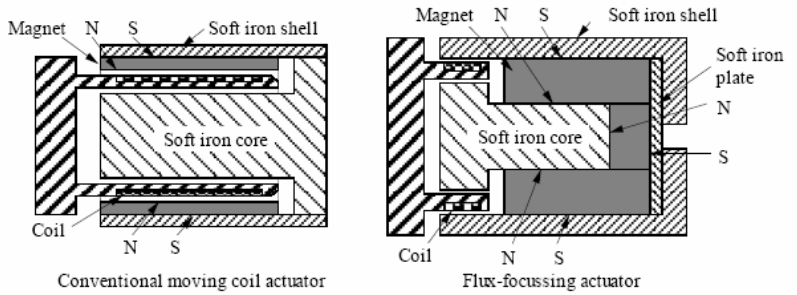
- Linear motor moving coil mounted to the inside of an air bearing carriage that rides on a T-shaped rail that is fully supported along its length



Source: Alexander Slocum, *Precision Machine Design*

Voice Coils

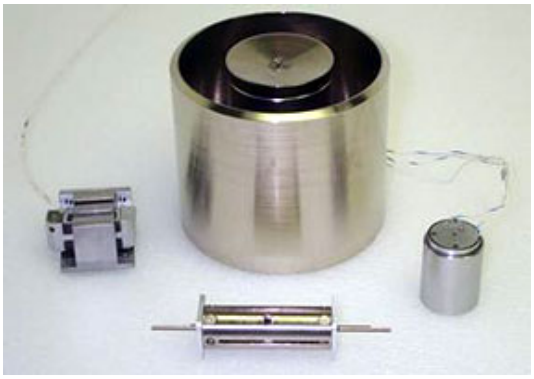
- Ideal for short strokes
- No phase switching circuitry required
- Can be used to drive fast tool servos (e.g., for piston turning machines)



Source: Alexander Slocum, *Precision Machine Design*

Voice Coils (contd.)

- Typical configurations:



Source: H2W Technologies

Types of Linear Motors

Types	Axial force F_a		Attractive force
	Rare earth	AlNiCo	
Voice coils	2	1	0
Stepping motors	1		5-10 x F_a
Brushless DC motors			
- Ironless core	2	1	0
- Iron core	2	1	5–10 x F_a

Source: Alexander Slocum, *Precision Machine Design*

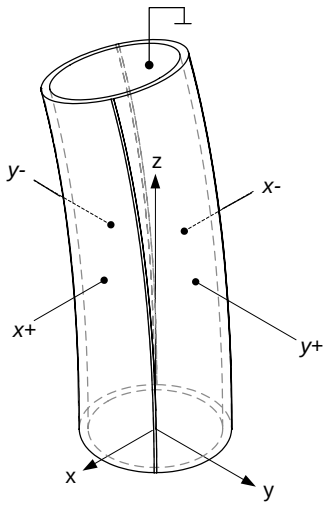
Piezoelectric Actuators

- Sub-nanometer resolution
- Large force generation (> 10,000 N)
- Sub-millisecond response
- No backlash, stiction or friction
- No magnetic fields
- Extremely low steady-state power consumption
- No wear and tear
- Vacuum and clean room compatibility
- Operation at cryogenic temperatures
- Variety of stacked PZT translators with typical displacement ranges of 5 to 200 µm

Source: Physikinstrumente

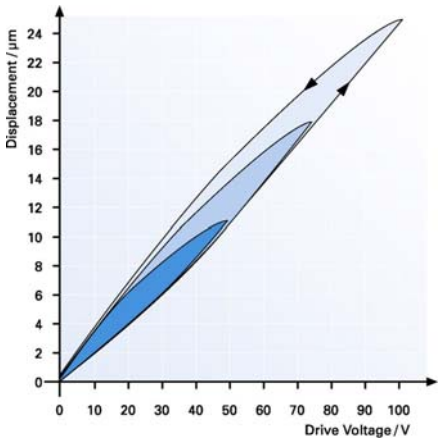
PZT Principle

- PZT are made from a ceramic material (crystal) that changes dimensions when a voltage is applied



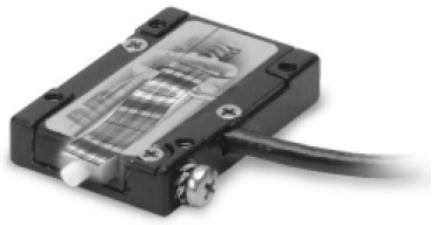
But

- Hysteresis
- High actuation voltage >100 V
- Aging
  - Aging refers to reduced piezo gain, among other things as a result of the de-poling process
  - Aging can be an issue for sensor or charge-generation applications (direct piezo effect), but with actuator applications it is negligible because re-poling occurs every time a higher electric field is applied to the actuator material in the poling direction



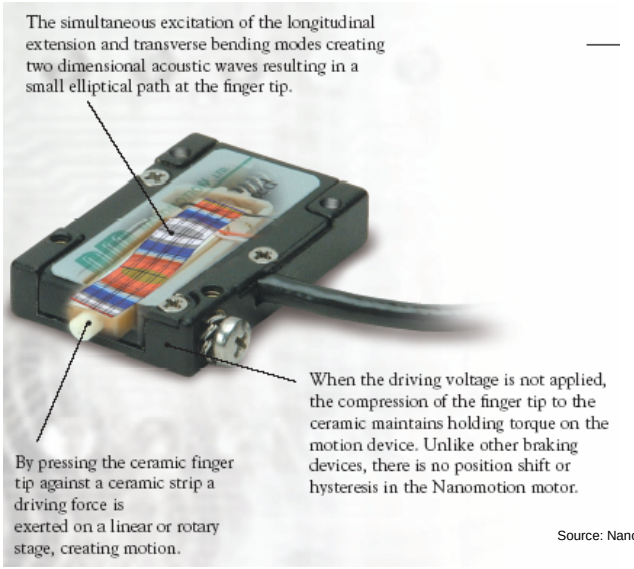
Piezo Linear Motor (Nanomotion)

- The Piezoelectric effect in piezo ceramics converts electrical field to mechanical strain
- Under special electrical excitation drive and ceramic geometry of Nanomotion motors, longitudinal extension and transverse bending oscillation modes are excited at close frequency proximity
- The simultaneous excitation of the longitudinal extension mode and the transverse bending mode creates a small elliptical trajectory of the ceramic edge, thus achieving the dual mode standing wave motor



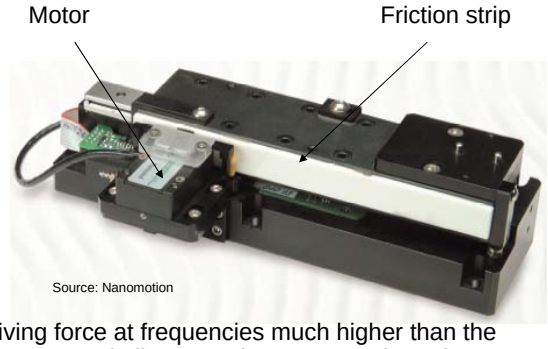
Source: Nanomotion

Piezo Linear Motor (Nanomotion)



Source: Nanomotion

Piezo Linear Motor (Nanomotion)

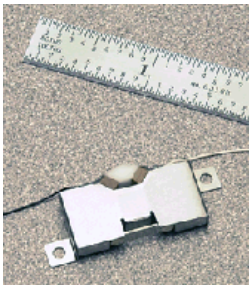


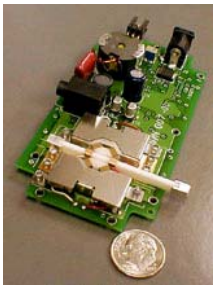
Source: Nanomotion

- By coupling the ceramic edge to a precision stage, a resultant driving force is exerted on the stage, causing stage movement
- Travel can be linear or rotary, depending on the coupling mechanism
- The periodic nature of the driving force at frequencies much higher than the mechanical resonance of the stage and allows continuous smooth motion for unlimited travel, while maintaining high resolution and positioning accuracy typical of piezoelectric devices
- While the driving voltage is not applied, the ceramic plate is stationary and generates holding torque on the stage
  - Holding torque of the Nanomotion motor does not cause any position shift

Piezo Linear Motor (EDO Ceramics)

- The piezoelectric motor operates using solid state piezoelectric ceramic crystals
- The piezoelectric effect exhibited by the ceramic crystals causes the ceramic to expand and contract as an electric field is applied across the polarized electrodes of the crystal
- The expansion and contraction under an alternating electric field causes a controlled, sinusoidal vibration
- This vibration drives the friction shoe that is coupled to flat friction strips, through friction, providing well controlled linear motion for the motor
- Each vibration cycle produces a small step, as small as a few nanometers
- The accumulation of these steps at a rate of 130,000 steps per second allows the motor to travel at speeds of up to 12 inches per second with a blocked force of up to 0.4 pounds

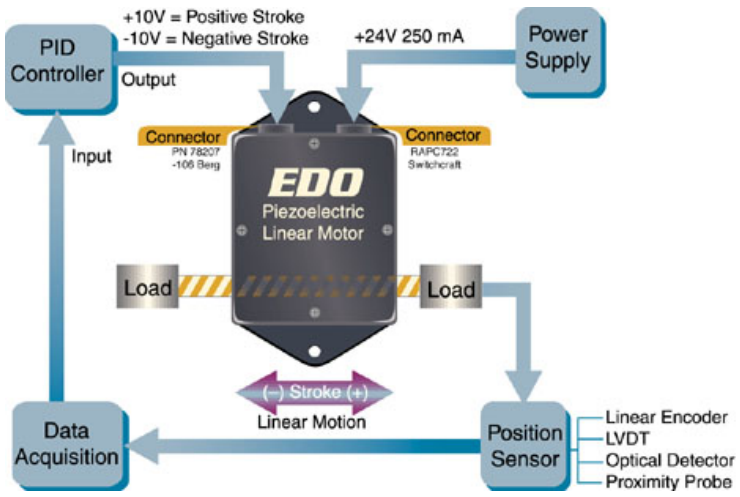




Source: EDO Ceramics

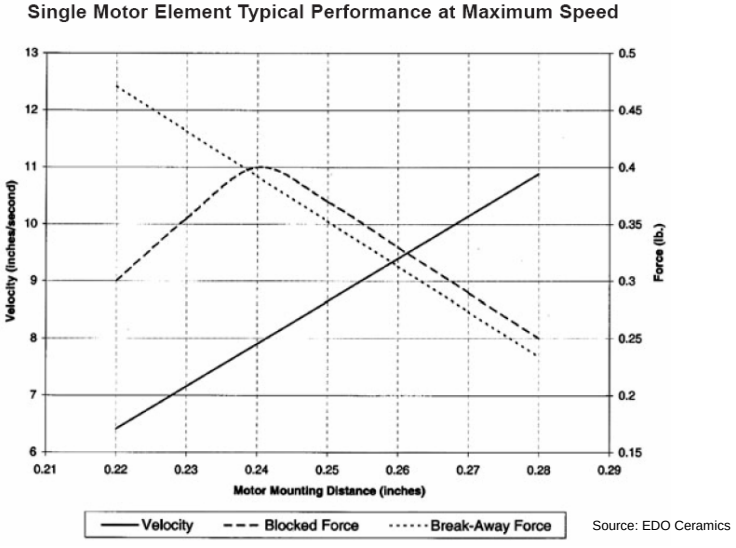
Flexure Head Video

Schematic of Servo Loop



Source: EDO Ceramics

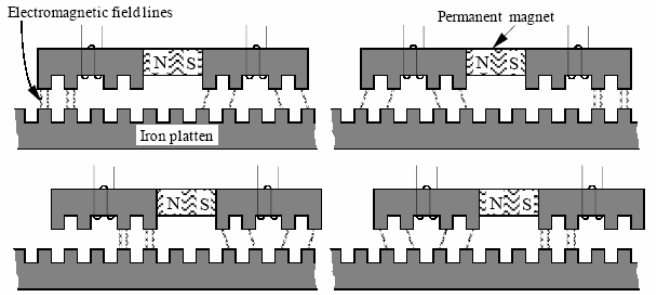
Piezo Linear Motor (EDO Ceramics)



Source: EDO Ceramics

Stepping Motors

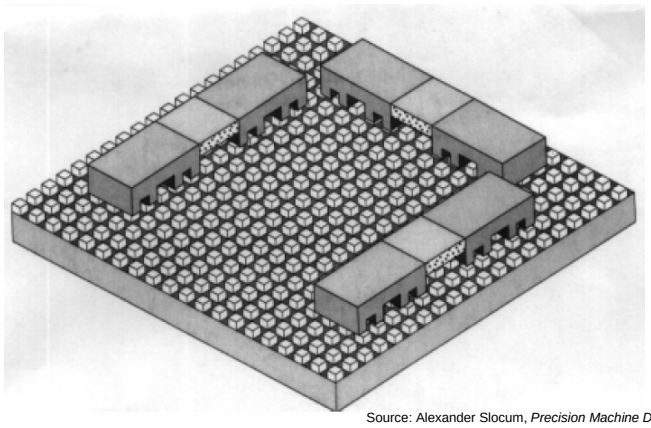
- Simple design
- Modest efficiency
- High attractive force requires robust bearings
- Microstepping can be used
- Closed loop servo mode, good to sensor resolution
- Current sequence in a Sawyer motor to produce linear motion:



Source: Alexander Slocum, *Precision Machine Design*

Sawyer Motor

- A two axis Sawyer motor (platen and air bearing to support the motor not shown).
  - Multiple forcers are used to control yaw.

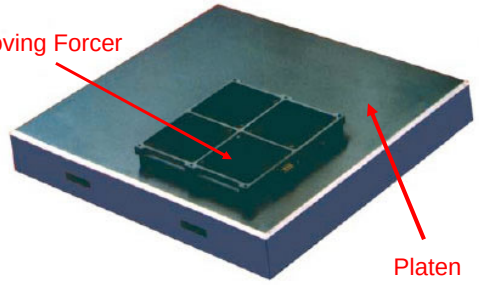


Source: Alexander Slocum, *Precision Machine Design*

Sawyer Motor

Moving Forcer Assembly:

- The forcer is made up of an aluminum housing that contains the motor windings, lamination stacks, and permanent magnets
- The active surface of the lamination is slotted to form teeth with a pitch of .040" [1 mm]
- The forcer comes in different widths and lengths, depending on the required force



- Multiple forcers can be supplied with a single platen to allow for independent moving heads

Platen:

- The platen is a precision ground steel plate that is slotted to form 0.02” x 0.02” [0.50x 0.50 mm] square teeth on the surface
- The spaces between the platen teeth are filled with epoxy to provide a flat air-bearing surface for the forcer
- The stroke of the forcer is a function of the length and width of the platen

Source: H2W Technologies

Sawyer Motor (contd.)

- Dual Axis Linear Stepper Motors are ideal for open loop positioning applications with light payloads
- Speeds up to 80 in/s [2 m/s] and strokes up to 40” x 60” [1 m x 1.5 m]
- Linear stepper motors are capable of very precise position, velocity and acceleration control when coupled with a microstepping drive and indexer
- The moving assembly called the “forcer” is supported by magnetically preloaded air-bearings imbedded in the active surface of the forcer between the forcer and platen
- The bearings support the payload and maintain the required .001” [0.025 mm] gap between the platen and the forcer
- The motion achieved with a full step is .010” [250 microns] and with a microstep it’s .00004” [1 micron]
- Integrating a linear encoder with the stepper motor provides a closed loop system

Source: H2W Technologies

Sawyer Motor - Advantages

- Low profile and small cross section
- High speed
- Low cost positioning stage solution
- No servo tuning required
- Multiple forcers on a single platen

Source: H2W Technologies

Sensor Systems

ME EN 7960 – Precision Machine Design
Topic 6

Outline

- Sensors and transducers
- Sensor performance characteristics
- Common analog output sensors
- Common digital sensors

Sensors and Transducers

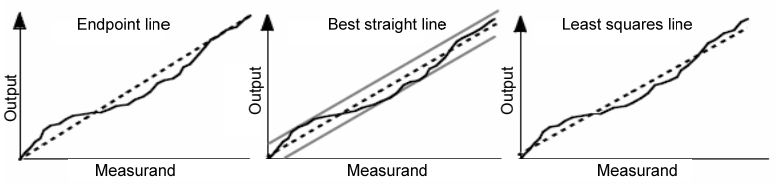
- A sensor is a device that responds to or detects a physical quantity and transmits the resulting signal to a controller
- A transducer is a sensor that converts (transduces) one form of energy to another form
- Basic types of sensors:
  - *Absolute:* The output is always relative to a fixed reference, regardless of the initial conditions
  - *Incremental:* The output is a series of binary pulses
    - Each pulse represents a change in the physical quantity by one resolution unit of the sensor
    - The pulses must be counted
  - *Analog:* The output is continuous and proportional to the physical quantity being measured
  - *Digital:* The output can only change by an incremental value given a change in the measured physical quantity

Sensor Performance Characteristics

- *Accuracy:* All sensors are accurate in that an input causes an output
  - The trick is to figure out what the sensor is saying
- *Averaged output:* Random errors can be reduced by the square root of the number of averages taken
- *Frequency Response:* is the effect of the physical quantity being measured as it varies in time, on the output of the sensor
- *Hysteresis* is the maximum difference in sensor output between measurements made from 0-100% full scale output (FSO), and 100-0% FSO

Sensor Performance Characteristics (contd.)

- *Linearity* is the variation in the constant of proportionality between the output signal and the measured physical quantity
- There are three different ways of fitting a straight line to the sensor's output versus input graph:
  - End point line
  - Best straight line
  - Least squares line



Source: Alexander Slocum, *Precision Machine Design*

Sensor Performance Characteristics (contd.)

  - The end point line connects the endpoints of the sensor's response curve
  - The best straight line is the line midway between the two parallel lines that completely envelop the sensor's response curve
  - The least squares line is the line drawn through the sensor's response curve such that the sum of the squares of the deviations from the straight line is minimized
- Mapping involves measuring the response of a sensor to a known input under known conditions
  - The results are then expressed in tabular or analytical form

Sensor Performance Characteristics (contd.)

- Most sensors' frequency responses are given in terms of the -3 db point
- If a sensor detects motion of a part and the output from the sensor used to control an axis to correct for the error:
  - The sensor should probably be operated well before its -3 db frequency response point
- The justification for this is:

Decibel (db)	Error
-0.0000087	1 ppm
-0.000087	10 ppm
-0.000869	100 ppm
-0.00869	1000 ppm
-0.087	1%
-0.915	10%
-3.0	30%

Sensor Performance Characteristics (contd.)

- The phase angle portion of the dynamic response:
  - Affects whether a sensor can be effectively used in a control system for a machine
- If there is too much lag:
  - It may not be possible for the mechanism to correct for errors sensed.
  - The error may have already irreversibly affected the process

Common Analog Output Sensors

- Capacitance sensors
- Hall effect sensors
- Inductive digital on/off proximity sensors
- Inductive distance measuring sensors
- Magnetic scales
- Magnetostrictive sensors
- Potentiometers
- Velocity sensors
- LVDT

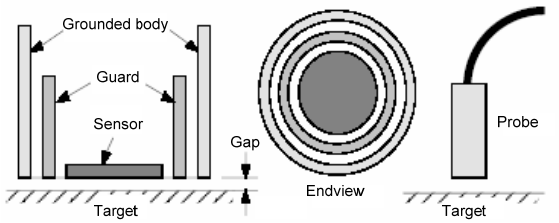
Capacitance Sensors

- Non-contact
- Requires clean, dry environment
- Resolution of 0.004% of full scale
- Bandwidth of up to 15 kHz
- Accuracy independent of target material (if conductive)
- Accuracy dependent of target material (if non-conductive)
- Target material has no minimum thickness
- Range 10 - 2000 microns (0.4 – 80 nanometer resolution @ 10 kHz)

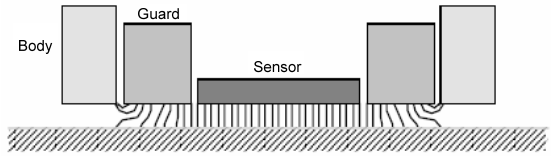
Source: Lion Precision

Capacitance Sensors

- General construction



- Field Properties

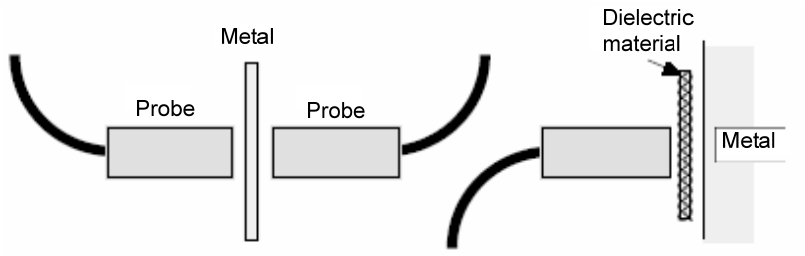


- Generally regarded as the most accurate type of analog limited range of motion sensor

Source: Alexander Slocum, *Precision Machine Design*

Capacitance Sensors (contd.)

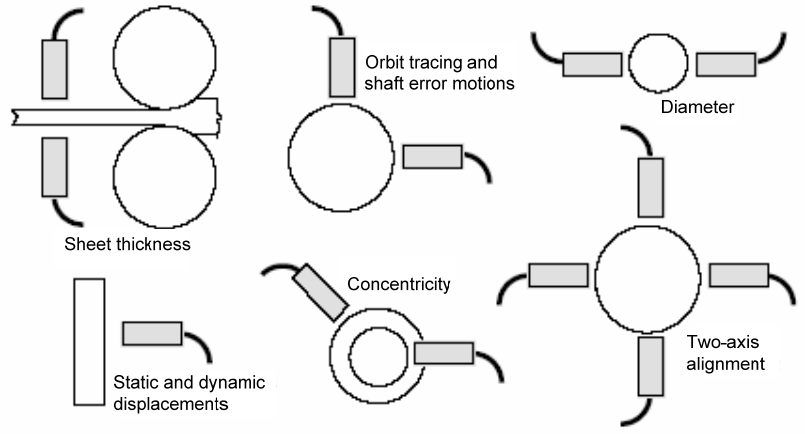
- Typical applications:
  - Position sensor for micropositioners
  - Material thickness sensing



Source: Alexander Slocum, *Precision Machine Design*

Capacitance Sensors (contd.)

– Metrology equipment (e.g. spindle error analyzers):



Source: Alexander Slocum, *Precision Machine Design*

Capacitive Sensors (contd.)

METRIC

Body Style	Sensor Size	Minimum Target Diameter (mm)	Extended Range Error Band DMT: 1.0%F.S.	Resolution nm		Standard Range Error Band DMT: 0.3%F.S. CD: 0.5%F.S.	Resolution nm				Fine Range Error Band DMT: 0.5%F.S. CD: 1.0%F.S. .	Resolution nm				Ultrafine Range Error Band DMT: 1.0%F.S.	Resolution nm	
				DMT,ECD			DMT,ECD		CD			DMT,ECD		CD			DMT,ECD	
			Range µm	RMS	P-P	Range µm	RMS	P-P	RMS	P-P	Range µm	RMS	P-P	RMS	P-P	Range µm	RMS	P-P
C1 C2	A	7.6	250 2250	60	600	500 1000	15	150	40	380	225 275	1.2	12	5.0	50	-	-	-
R1 C7 †	B	4.1	250 1500	50	500	250 750	20	160	34	340	125 175	2.5	25	5.0	50	-	-	-
	C	2.3	125 625	25	250	125 375	15	120	24	240	75 125	2.5	25	5.0	50	20 30	0.4	4.0
C3 C4	D	2.5	125 625	20	200	125 375	10	100	24	240	75 125	2.2	22	5.0	50	20 30	0.4	4.0
C5 C6	E	1.0	-	-	-	100 200*	25	250	50	500	75 100*	5.0	50	15.0	100	-	-	-
C11 C12	F	0.5	60 140*	9	70	50 100*	8	60	-	-	20 30*	1.5	10	-	-	-	-	-
A1 C8	G	7.6	500 2500	60	600	1000 2000	50	500	100	1000	-	-	-	-	-	-	-	-
R2	H	4.1	3000 13000*	250	2500	5000 7500	100	1000	300	2000	-	-	-	-	-	-	-	-

Source: Lion Precision

Inductive Probes

- Use the principle of Eddy currents for sensing distance
- Non-contact.
- Resolution 0.008% of full scale
- Bandwidth up to 80 kHz
- Very sensitive to material properties (ferrous vs. non-ferrous)
- Insensitive to environment
- Target material must be conductive
- Target must have minimum thickness (0.01 – 1.6mm)
- Range 500 – 15,000 microns (resolution 0.04 – 1.2 microns)

Source: Lion Precision

Inductive Probes

Metric Chart (Change to Inch Chart)

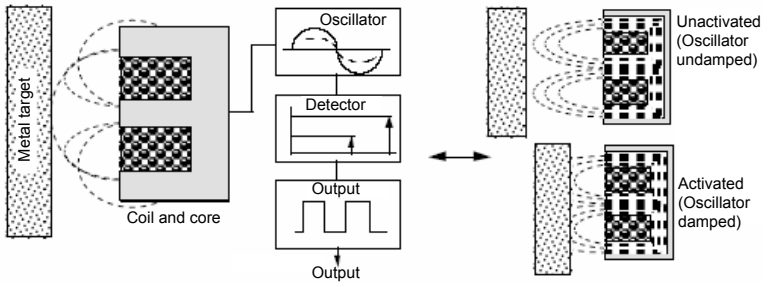
Range and Offset are in millimeters, Resolution is in micrometers

Probe Model	Calibration	Range (mm)	Offset (mm)	Resolution (µmeter)		
				1kHz	10kHz	80kHz
U3	Standard	0.50	0.05	0.02	0.04	0.30
U5	Standard	1.25	0.25	0.05	0.10	0.75
U8	Standard	2.00	0.35	0.08	0.16	1.20
U12	Standard	3.50	0.60	0.14	0.28	2.10
U18	Standard	5.00	0.75	0.20	0.40	3.05
U25	Standard	8.00	1.25	0.28	0.57	4.80
U38	Standard	12.5	1.50	0.46	0.97	7.16
U50	Standard	15.0	2.00	0.66	1.32	9.90

Source: Lion Precision

Inductive Digital On/Off Proximity Sensors

- General operating principle (Courtesy of Turck Inc.):

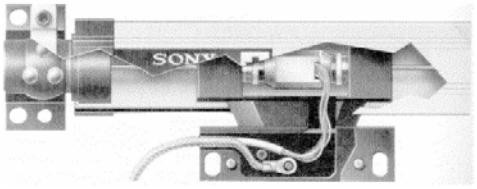


Source: Alexander Slocum, *Precision Machine Design*

- Typical applications:
  - Industrial limit switches
  - "Coarse" home position sensor for machine tools (fine home position via encoder home pulse)

Magnetic Scales

- Operates on same principle that allows a disk drive to locate stored information
- More robust than linear optical encoders
- Magnetically encoded linear scale and sliding read head (Courtesy of Sony Magnescale Inc.):

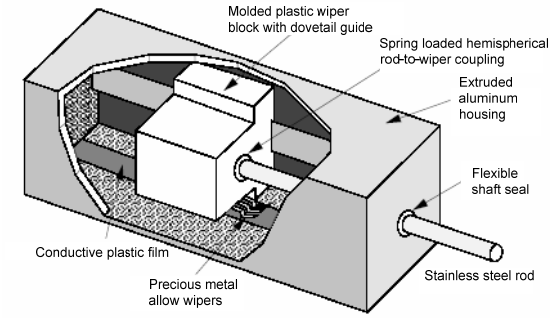


- Becoming more and more common sensor for measuring linear motion of machine tool axes

Source: Alexander Slocum, *Precision Machine Design*

Potentiometers

- General operating principle (Courtesy of Vernitech Corp.):



- Typical applications:
  - As a sensor in a high reliability all analog servo system
  - Short range of motion servo systems

Source: Alexander Slocum, *Precision Machine Design*

Linear Variable Displacement Transducers (LVDT)

- DC-operated ("DC-to-DC") LVDT's are easy to install and require little signal conditioning, and relatively inexpensive
- AC-operated LVDT's are generally smaller in body size and more accurate than DC versions
  - They will also normally operate at higher temperatures
- Range 1 – 940 mm (accuracy 0.25 – 235 microns)
- Submersible (some models)

Source: Sensotec

Velocity Sensors

- Linear velocity sensors tend to act like antennas, so they pick up EMI easily; thus their use should be avoided
- Rotary velocity sensors (tachometers) are essentially driven DC motors
- Typical applications:
  - Speed control
  - Analog velocity feedback

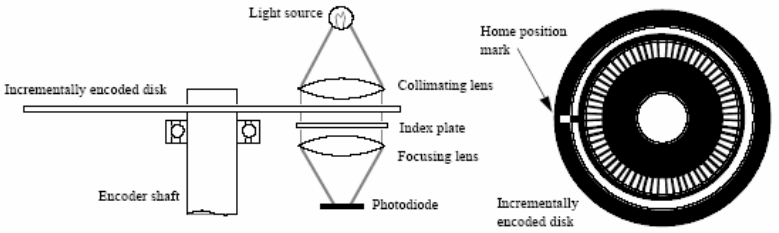
Common Optical Sensors

- Autocollimators
- Optical encoders
- Fiber optic sensors
- Interferometric sensors
- Laser triangulation sensors
- Vision systems

Optical Encoders

- Common types:
  - Incremental position encoders
  - Interpolation (E.G., Moire fringe) encoders
  - Absolute position encoders
  - Diffraction encoders
  - Quadrature logic
  - Typical characteristics of optical encoders

Incremental Position Encoders



Source: Alexander Slocum, *Precision Machine Design*

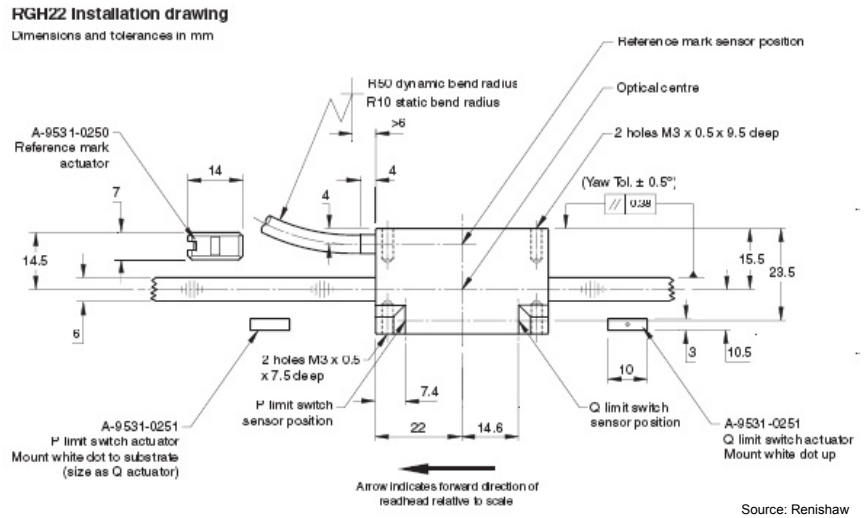
- Most commonly used type
- Reasonable resolution
- Inexpensive and widely available
- Quality is typically proportional to price
  - Poor gratings and poor electronics lead to output signal orthogonality errors
  - Quadrature signals are 90°±N°, which cause velocity calculation errors in control loops

Optical Encoder (Renishaw)

Source: Renishaw

- Non-contact
- No enclosure
- Resolution 50 nm – 5 microns
- Built-in interpolation
  - Robust against EMF
- Built-in limit and reference switch
- Uses flexible, self-adhesive 20 micron tape
- Maximum speed 10 – 3 m/s
- $800-1000 for read head, $450 for tape (3m)

Optical Encoder (Renishaw)



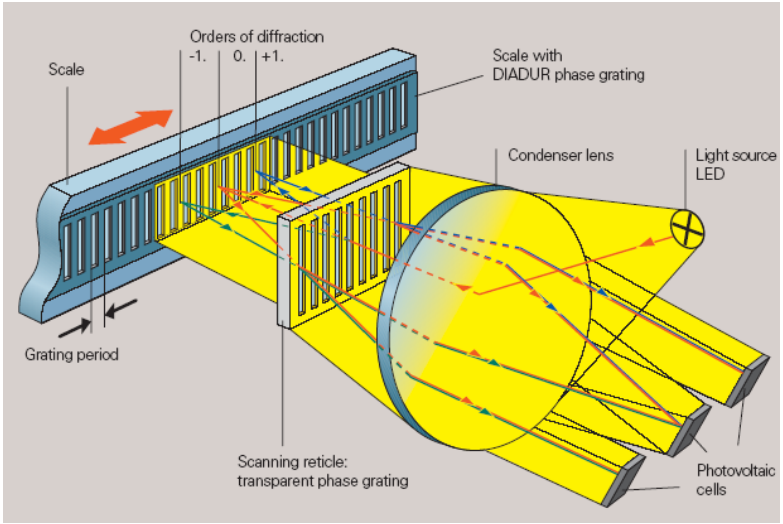
Source: Renishaw

Diffraction Encoders

- The interferential scanning principle exploits the diffraction and interference of light on a fine graduation to produce signals used to measure displacement
- A step grating is used as the measuring standard: reflective lines 0.2 µm high are applied to a flat, reflective surface
- In front of that is the scanning reticle—a transparent phase grating with the same grating period as the scale
- When a light wave passes through the scanning reticle, it is diffracted into three partial waves of the orders –1, 0, and +1, with approximately equal luminous intensity
  - The waves are diffracted by the scale such that most of the luminous intensity is found in the reflected diffraction orders +1 and –1
- These partial waves meet again at the phase grating of the scanning reticle where they are diffracted again and interfere
- This produces essentially three waves that leave the scanning reticle at different angles
- Photovoltaic cells convert this alternating light intensity into electrical signals

Source: Heidenhain

Diffraction Principle



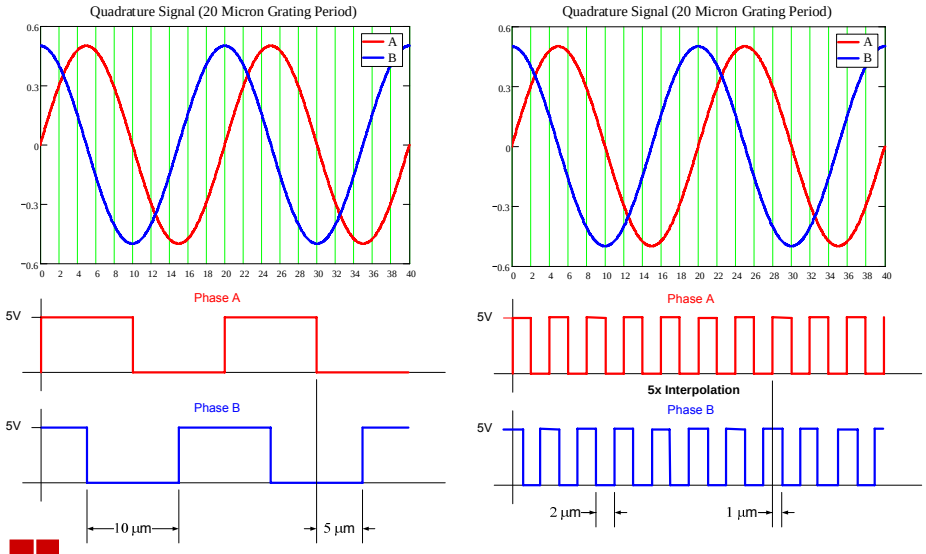
Source: Heidenhain

Optical Encoder (Heidenhain)

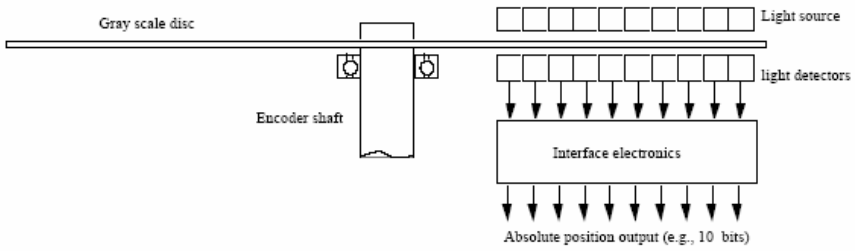
- Non-contact
- No enclosure
- 70 – 270 mm measuring length
- 0.001 micron resolution
- Accuracy ± 0.5 µm
- Glass scale

Source: Heidenhain

Quadrature Encoder (20 µm Grating Period)



Absolute Position Encoder

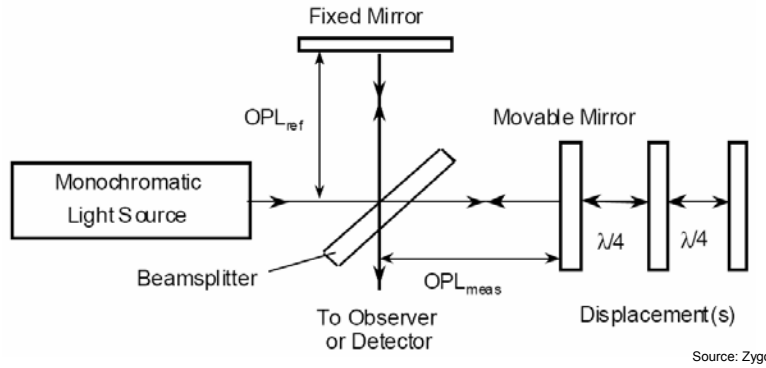


Source: Alexander Slocum, *Precision Machine Design*

- Not commonly used on machine tools because most have to be reset upon startup anyway
- Moderate resolution for a price

Interferometers

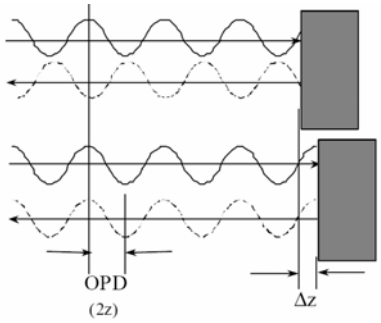
- Michelson interferometers count fringes which limits the resolution to about $\lambda/8$
- Heterodyne techniques can be used to achieve two orders of magnitude greater resolution:



Source: Zygo

Principle of Interferometers

- **Scenario 1:** The incoming and the reflected beam are shifted by 180 degrees -> intensities cancel each other
- **Scenario 2:** The incoming and reflected beam are in phase -> intensities add up
- When two waves interfere, the independent electric (E) fields sum to generate a net E field
  - The Intensity (I) is equal to the square net E field

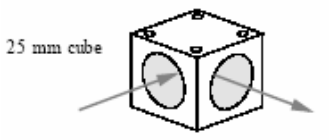


- If waves intersect in phase, totally constructive interference occurs
  - This is the condition of maximum brightness
- When the waves are 180° out of phase, totally destructive interference occurs
  - This is the condition of complete extinction.
- When the target is displaced by half of the wavelength, the intensity goes through exactly one period
  - This corresponds to a light-dark-light transition

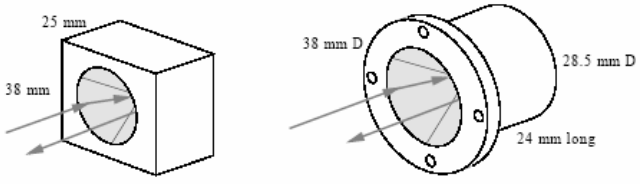
Source: Zygo

Beam Handling Components

- Beambender: A plane mirror:



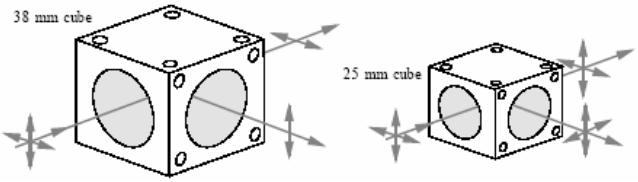
  - Careful to make the bend 90° to avoid polarization leakage problems.
- Linear retroreflectors: Return light parallel to its incoming path:



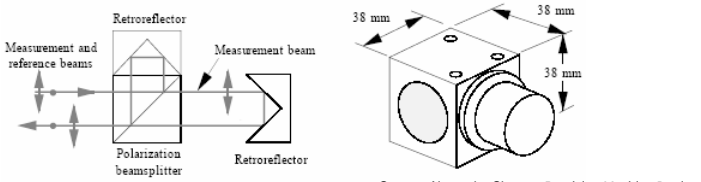
Source: Alexander Slocum, *Precision Machine Design*

Beam Handling Components (contd.)

- Beamsplitter: Separates orthogonally polarized beams into two components:

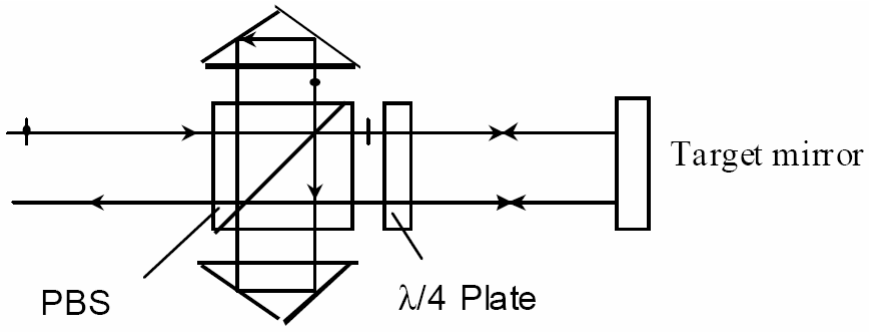


- Linear displacement interferometer: Combines polarization beamsplitter and a retroreflector:



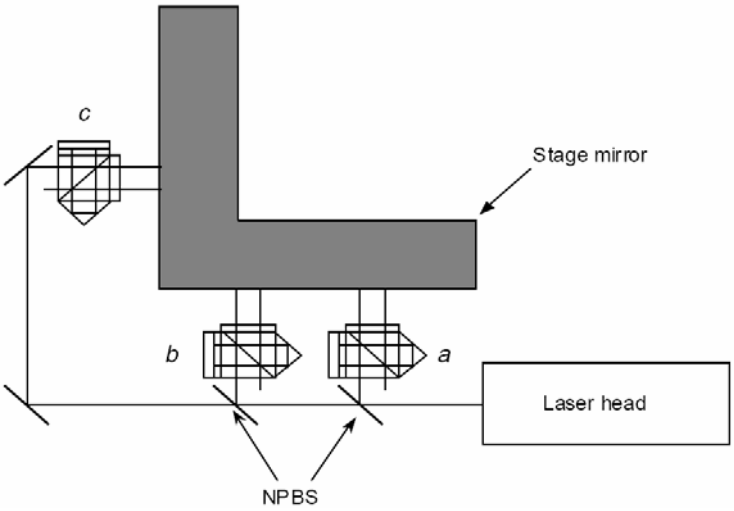
Source: Alexander Slocum, *Precision Machine Design*

Plane Mirror Interferometer



Source: Zygo

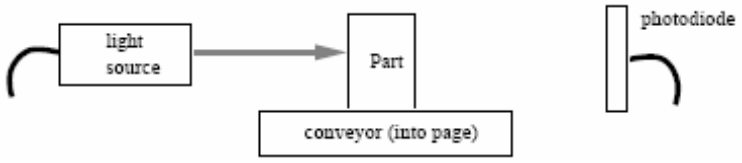
Wafer Stage Metrology



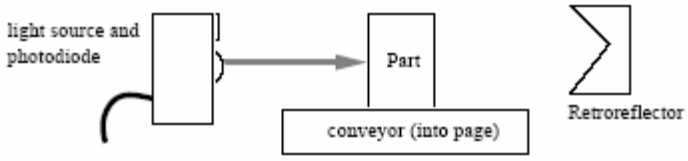
Source: Zygo

Photoelectric Transducers

- Opposed mode (interrupted beam) operation of a photoelectric proximity sensor:



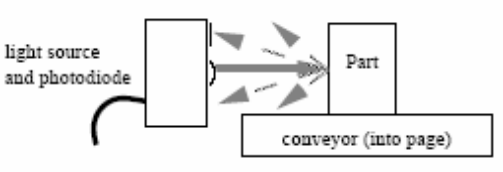
- Retroreflective mode operation of a photoelectric proximitv sensor:



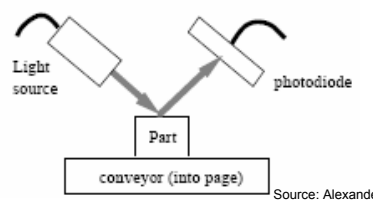
Source: Alexander Slocum, *Precision Machine Design*

Photoelectric Transducers (contd.)

- Diffuse reflection mode operation of a photoelectric proximity sensor



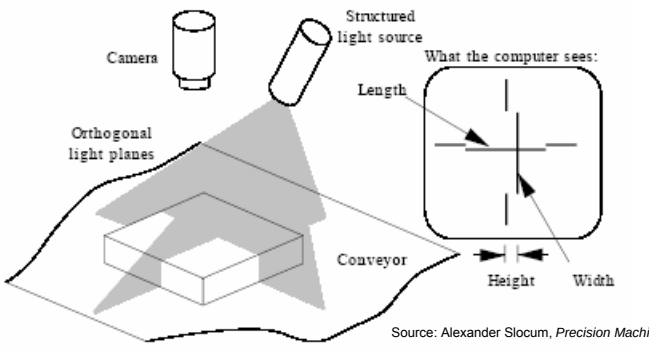
- Specular reflection mode operation of a photoelectric proximity sensor:



Source: Alexander Slocum, *Precision Machine Design*

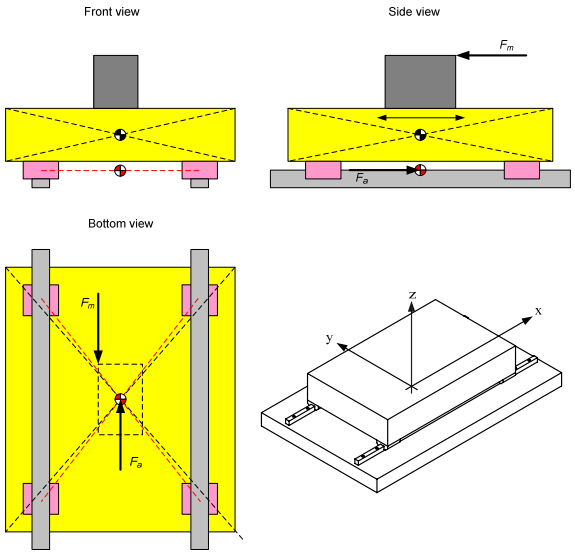
Vision Systems

- Perform well if they know what they are looking for:
  - Optical comparators
  - Mapping the shape of a tool
  - Measuring part dimensions using structured light (After Landman):

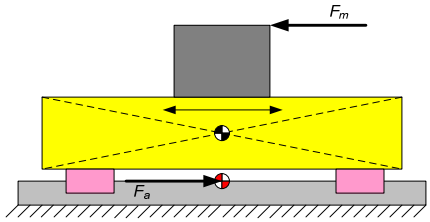


Source: Alexander Slocum, *Precision Machine Design*

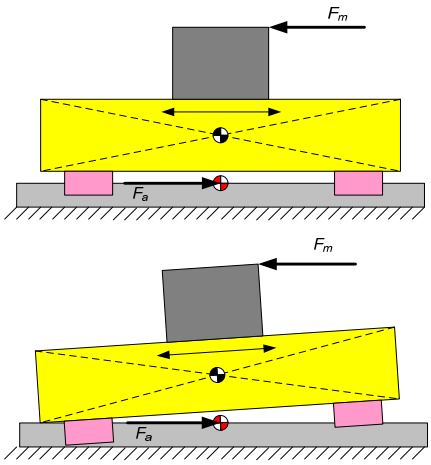
Sensor Positioning



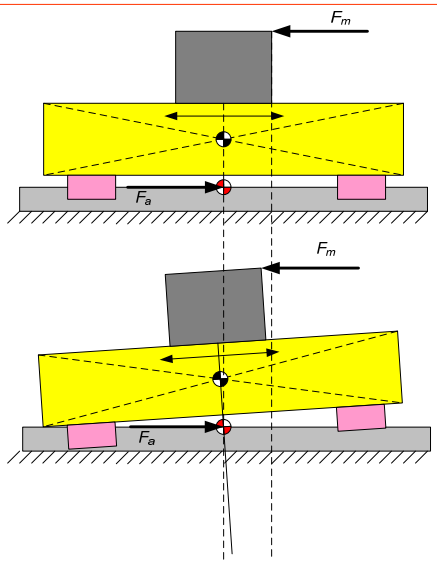
Vertical Sensor Position



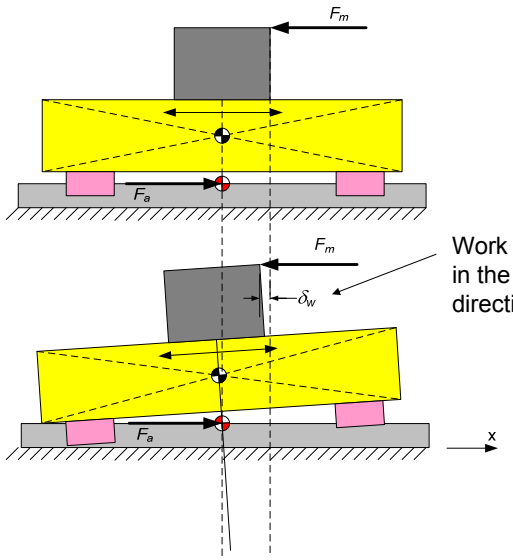
Vertical Sensor Position (contd.)



Vertical Sensor Position (contd.)

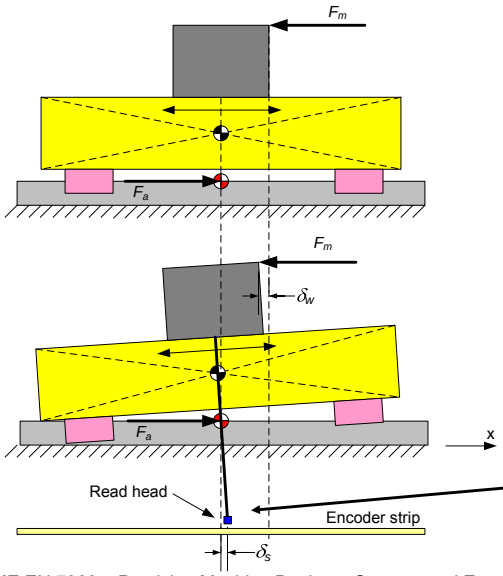


Vertical Sensor Position (contd.)



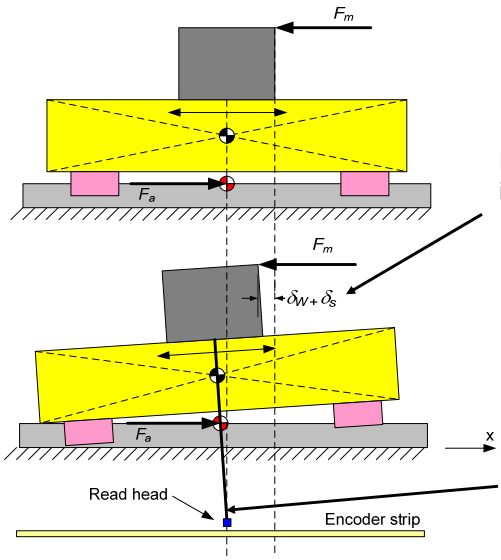
Work piece is pushed in the negative x-direction

Vertical Sensor Position (contd.)



Sensor below the center of stiffness is pushed in the positive x-direction

Vertical Sensor Position (contd.)



Error of work piece is now the sum of δ_w *and* δ_s

Control system regains target position by moving axis by $-\delta_s$

Conclusion

- The position of the sensor is critically important to the accuracy of a design
- As a rule of thumb, the sensor should be positioned as close as possible to the center of stiffness, but NEVER below

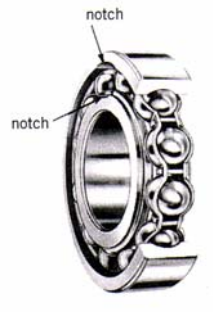
Contact Stresses and Deformations

ME EN 7960 – Precision Machine Design
Topic 7

Curved Surfaces in Contact

- The theoretical contact area of two spheres is a point (= 0-dimensional)
- The theoretical contact area of two parallel cylinders is a line (= 1-dimensional)
  - → As a result, the pressure between two curved surfaces should be infinite
  - → The infinite pressure at the contact should cause immediate yielding of both surfaces
- In reality, a small contact area is being created through elastic deformation, thereby limiting the stresses considerably
- These contact stresses are called Hertz contact stresses

Curved Surfaces in Contact – Examples



Rotary ball bearing

Rotary roller bearing

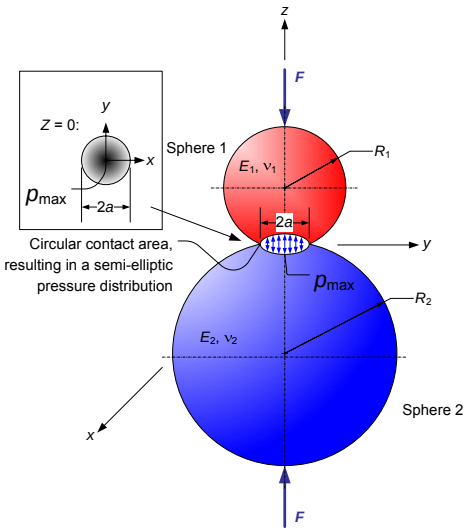
Linear bearings (ball and rollers)

Curved Surfaces in Contact – Examples (contd.)

Ball screw

Gears

Spheres in Contact



The radius of the contact area is given by:

$$a = \sqrt[3]{\frac{3F\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]}{4\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

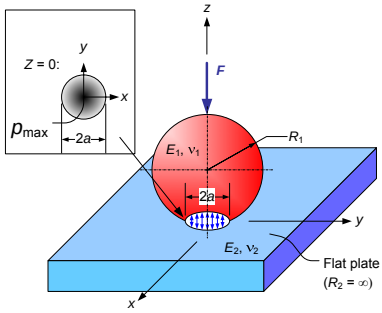
Where E_1 and E_2 are the moduli of elasticity for spheres 1 and 2 and ν_1 and ν_2 are the Poisson's ratios, respectively

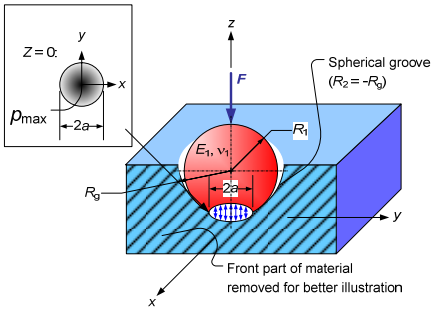
The maximum contact pressure at the center of the circular contact area is:

$$p_{max} = \frac{3F}{2\pi a^2}$$

Spheres in Contact (contd.)

- The equations for two spheres in contact are also valid for:
  - Sphere on a flat plate (a flat plate is a sphere with an infinitely large radius)
  - Sphere in a spherical groove (a spherical groove is a sphere with a negative radius)





Spheres in Contact – Principal Stresses

The principal stresses σ_1, σ_2, and σ_3 are generated on the z-axis:

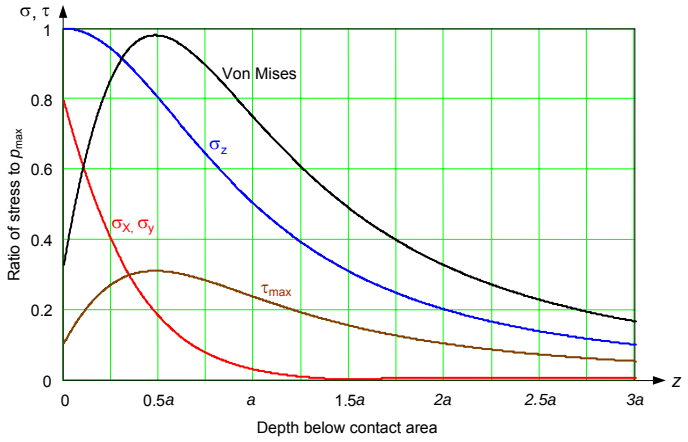
$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[(1+\nu)\left(1-\left|\frac{z}{a}\right|\arctan\left|\frac{a}{z}\right|\right)-\frac{1}{2\left(\frac{z^2}{a^2}+1\right)}\right]$$

$$\sigma_3 = \sigma_z = -p_{\max}\left(\frac{z^2}{a^2}+1\right)^{-1}$$

The principal shear stresses are found as:

$$|\tau_1| = |\tau_2| = \tau_{\max} = \left|\frac{\sigma_1-\sigma_3}{2}\right| \qquad |\tau_3| = 0$$

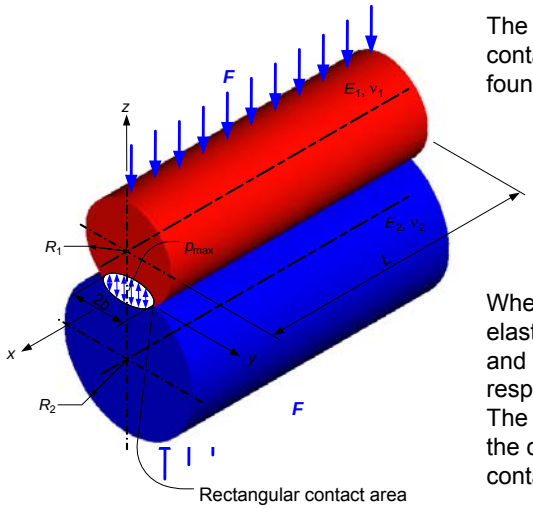
Spheres in Contact – Vertical Stress Distribution at Center of Contact Area



Plot shows material with Poisson's ratio ν = 0.3

- The maximum shear and Von Mises stress are reached below the contact area
- This causes pitting where little pieces of material break out of the surface

Cylinders in Contact



Rectangular contact area with semi-elliptical pressure distribution

The half-width b of the rectangular contact area of two parallel cylinders is found as:

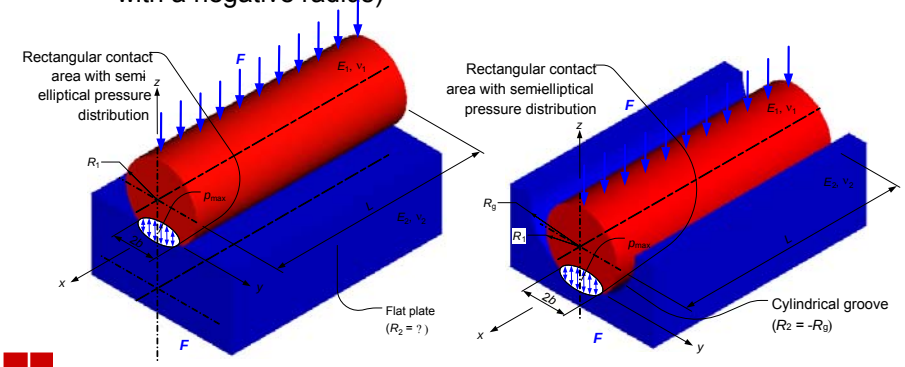
$$b = \sqrt{\frac{4F\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]}{\pi L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

Where E_1 and E_2 are the moduli of elasticity for cylinders 1 and 2 and ν_1 and ν_2 are the Poisson's ratios, respectively. L is the length of contact. The maximum contact pressure along the center line of the rectangular contact area is:

$$p_{\max} = \frac{2F}{\pi b L}$$

Cylinders in Contact (contd.)

- The equations for two cylinders in contact are also valid for:
  - Cylinder on a flat plate (a flat plate is a cylinder with an infinitely large radius)
  - Cylinder in a cylindrical groove (a cylindrical groove is a cylinder with a negative radius)



Cylinders in Contact – Principal Stresses

The principal stresses σ_1, σ_2, and σ_3 are generated on the z-axis:

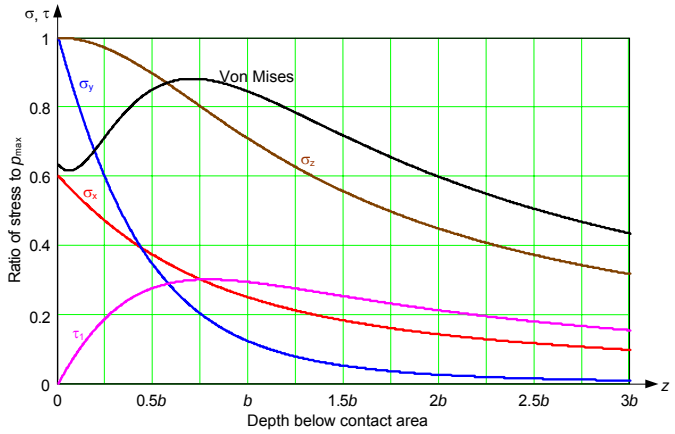
$$\sigma_1 = \sigma_x = -2\nu p_{\max}\left[\sqrt{\frac{z^2}{b^2}+1} - \left|\frac{z}{b}\right|\right]$$

$$\sigma_2 = \sigma_y = -p_{\max}\left[\left(2-\left(\frac{z^2}{b^2}+1\right)^{-1}\right)\sqrt{\frac{z^2}{b^2}+1} - 2\left|\frac{z}{b}\right|\right]$$

$$\sigma_3 = \sigma_z = -p_{\max}\left[\sqrt{\frac{z^2}{b^2}+1}\right]^{-1}$$

$$\tau_1 = \left|\frac{\sigma_2-\sigma_3}{2}\right|, \quad \tau_2 = \left|\frac{\sigma_1-\sigma_3}{2}\right|, \quad \tau_3 = \left|\frac{\sigma_1-\sigma_2}{2}\right|$$

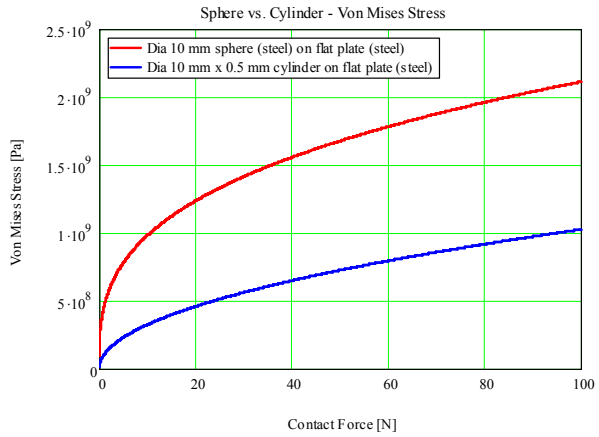
Cylinders in Contact – Vertical Stress Distribution along Centerline of Contact Area



Plot shows material with Poisson's ratio $\nu = 0.3$

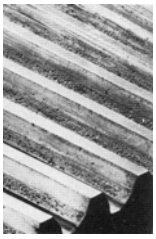
- The maximum shear and Von Mises stress are reached below the contact area
- This causes pitting where little pieces of material break out of the surface

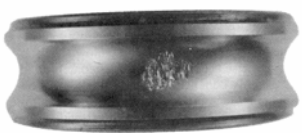
Sphere vs. Cylinder – Von Mises Stress



- The Von Mises stress does not increase linearly with the contact force
- The point contact of a sphere creates significantly larger stresses than the line contact of a cylinder

Effects of Contact Stresses - Fatigue





Elastic Deformation of Curved Surfaces

The displacement of the centers of two spheres is given by:

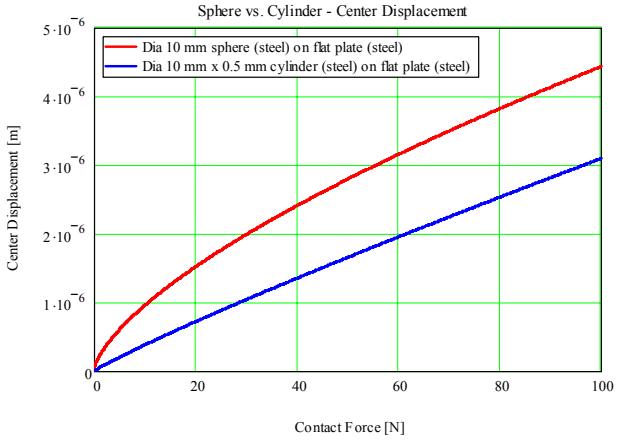
$$\delta_s = 1.04\left[F\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)\right]^{2/3}\left[\frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{1/3}$$

The displacement of the centers of two cylinders is given by:
With $\nu_1 = \nu_2 = \nu$, and $E_1 = E_2 = E$:

$$\delta_c = \frac{2F\left(1-\nu^2\right)}{\pi LE}\left(\frac{2}{3}+\ln\frac{4R_1}{b}+\ln\frac{4R_2}{b}\right)$$

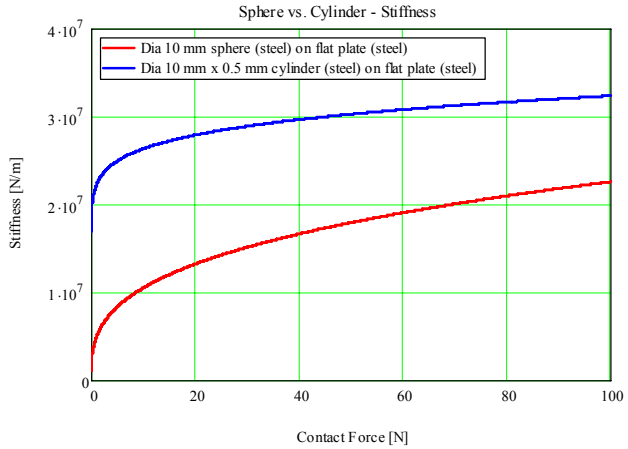
Note that the center displacements are highly nonlinear functions of the load

Sphere vs. Cylinder – Center Displacement



- The point contact of a sphere creates significantly larger center displacements than the line contact of a cylinder

Sphere vs. Cylinder – Stiffness

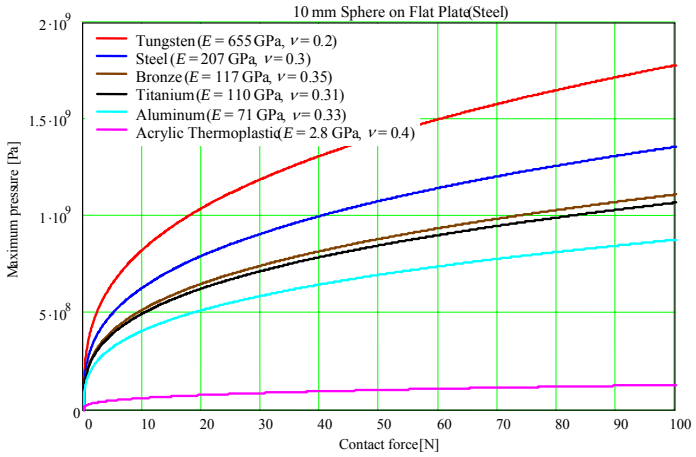


- The point contact of a sphere creates significantly lower stiffness than the line contact of a cylinder

Effects of Material Combinations

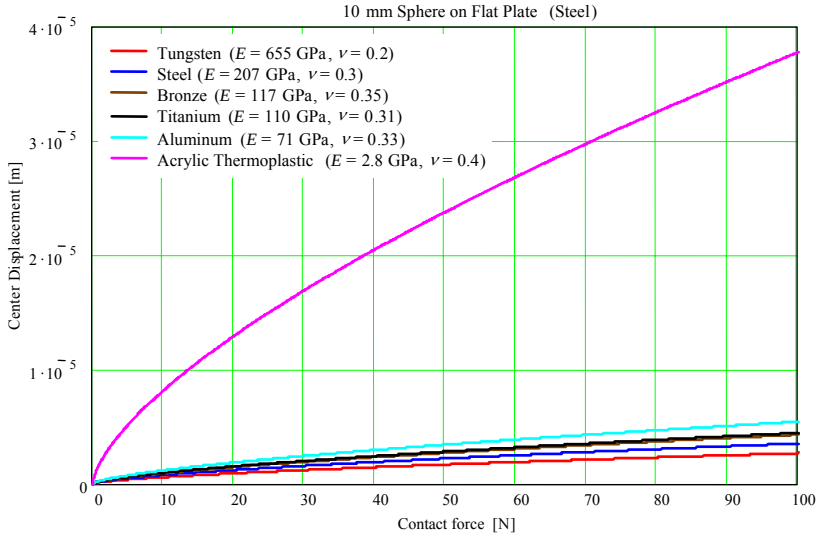
- The maximum contact pressure between two curved surfaces depends on:
  - Type of curvature (sphere vs. cylinder)
  - Radius of curvature
  - Magnitude of contact force
  - Elastic modulus and Poisson's ratio of contact surfaces
- Through careful material pairing, contact stresses may be lowered

Contact Pressure Depending on Material Combination



- Materials with a lower modulus will experience larger deformations, resulting in a lower contact pressure

Center Displacement Depending on Material Combination



Linear Rolling Bearings

ME EN 7960 – Precision Machine Design
Topic 8

Bearings

- Bearings are mechanical elements that free one or more of the six degrees of freedom while constraining the remainders
- Bearings can be classified in two main ways
  1. Degrees of freedom (linear, rotary)
  2. Contact yes or no (contact, non-contact)

Non-Contact Bearings

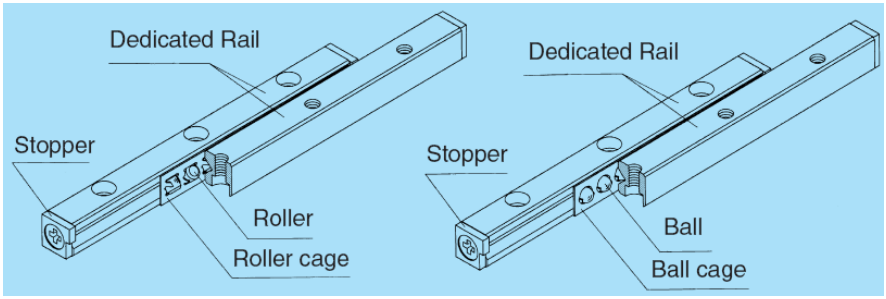
- Non-contact bearings maintain a gap between the moving surfaces, thereby avoiding any kind of contact
- The gap is maintained through:
  - Pressurized medium (air, water, oil)
  - External forces (magnetic forces)
- Non-contact bearings are used in precision machinery where accuracy and speed requirements prohibit the use of contact bearings
  - Hydrostatic: high-speed machining spindles
  - Aerostatic: slides for coordinate measuring machines (CMM), dental drills. Also: read head of hard disks.
  - Magnetic bearings: high-speed trains (Transrapid)

Contact Bearings

- Contact bearings can be grouped in two main categories according to their type of contact:
  1. Rolling contact bearings
  2. Sliding contact bearings
- Rolling contact bearings separate the mechanical components through the use of balls or rollers
- Sliding contact surfaces make contact, either with or without lubrication

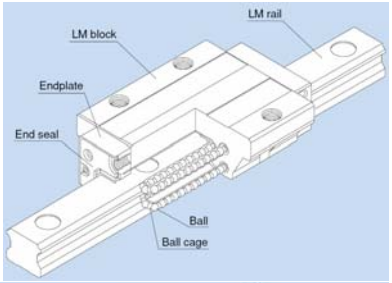
Basic Types

- There are three main types of rolling element linear motion bearings:
  - Non-recirculating balls or rollers
  - Recirculating balls
  - Recirculating rollers



Source: THK

Recirculating Linear Bearings



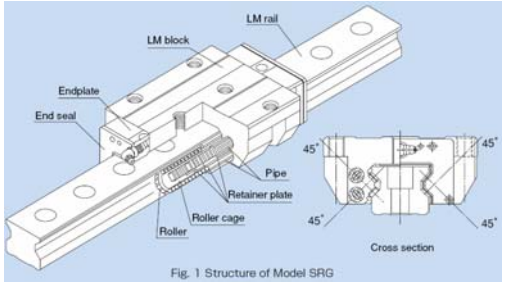


Fig. 1 Structure of Model SRG

Source: THK

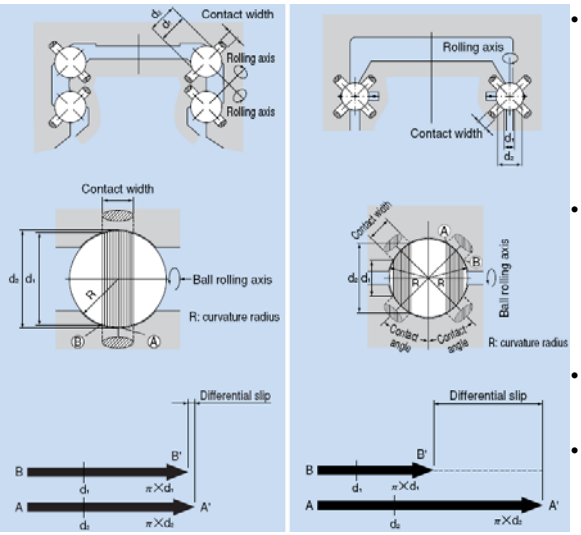
General Design Considerations

- Before choosing a rolling element linear motion bearing, there are several fundamental issues to consider including:
  - Balls or rollers, which to use
  - Shape of the contact surface
  - To recirculate or not to recirculate
  - Bearing spacing
- Bear in mind many of the fine points of general characteristics of rotary motion bearings

Balls or Rollers?

- Balls can be made to higher accuracies
- Balls have no potential to skid sideways
- Rollers typically have to have a slight barrel shape (or a slightly curved raceway) to avoid edge loading
- Rollers can have greater load capacity than balls in a circular arch
- In the end, all contacts are governed by the Hertz equations, and physics rules over sales talk
  - Look at the specification sheets
  - Look at straightness data and rolling element noise spectrums
  - Build and test a system if necessary
- The wise user selects interchangeable components!

Circular groove vs. gothic arch



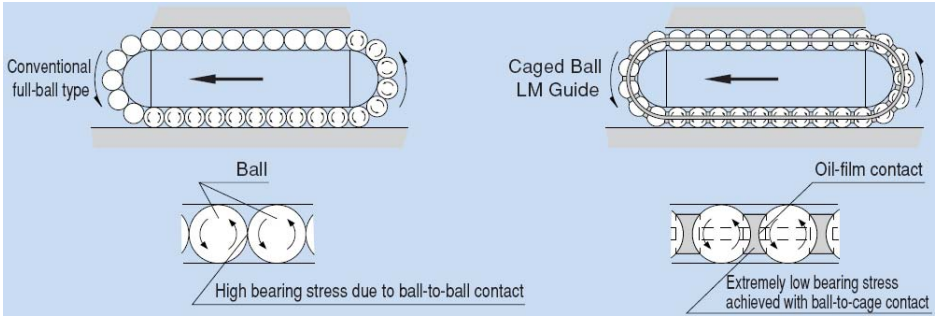
- When the ball rotates one revolution, the ball slips by the difference between the circumference of the inner contact area (πd_1) and the of the outer diameter (πd_2)
- If the difference is large, the ball rotates while slipping and the friction coefficient increases up to 10 times, causing increases resistance and wear
- Circular grooves have 3% slip, gothic arches up to 40%
- Gothic arces have larger contact area
  - More damping

Source: THK

To Recirculate or to Not Recirculate

- Recirculating elements allow for "infinite" travel
- As the elements leave the raceway and enter the raceway, they generate acoustical and straightness noise
- In most bearings, the elements are not retained, so they can rub on each other causing friction and noise
  - THK's new patented NR series encapsulates the balls in a polymer necklace that keeps the balls spaced, and helps to keep them lubricated

THK NR Series



- This reduces rolling element noise by 50%
- This increases maximum speed to up to 4 m/s

- Recirculating bearings are often compact and can resist loads and moments from all directions
- In general, for short stroke precision applications, it is often best to use non-recirculating bearings

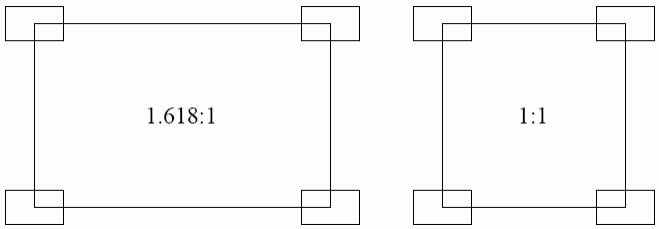
Source: THK

Bearing Spacing

- For machine tools, typically the system will be over-constrained anyway
  - One should not always be shy about supporting a carriage at all four corners
- The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
  - The smoother the linear motion will be and the less the chance of walking (yaw error)
- First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1
- For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1)

Bearing Spacing (contd.)

- The minimum length to width ratio is 1:1 to minimize yaw error



- The higher the speed, the higher the length to width ratio should be
- For large moving bridge machines:
  - It is often necessary to use actuators and sensors on both sides of the bridge with one system slaved to the other

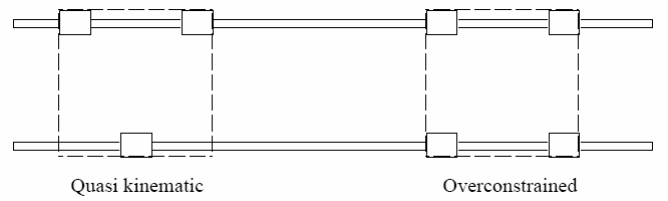
Source: Alexander Slocum, *Precision Machine Design*

Detailed Design Considerations

- Performance considerations
  - Running parallelism, repeatability, and resolution
  - Lateral and moment load support capability
  - Allowance for thermal growth
  - Alignment requirements
  - Preload and frictional properties
- Try to visualize forces and moments as "fluids" and see how they flow from the carriage to the bearing to the machine

Detailed Design Considerations

- For machine tool applications where high cutting forces and moments must be resisted:
  - One is virtually required to use an over-constrained bearing arrangement
  - With reasonable manufacturing tolerances, increased stiffness and elastic averaging effect can be beneficial:



Source: Alexander Slocum, *Precision Machine Design*

Design Considerations

- Speed and acceleration limits
  - < 60-120 m/min (2000-4000 ipm) and 1 g
  - At higher speeds, rapidly use up L_{100} life, and requires oil lubrication
- Applied loads
  - Large load capacity is achieved with many elements
    - Remember, load capacity quoted in a catalogs is usually for 100 km of travel
    - The load/life relation is cubic:

$$F_{design} = F_{100km}\left(\frac{L_{design,100km}}{100km}\right)^{-1/3}$$

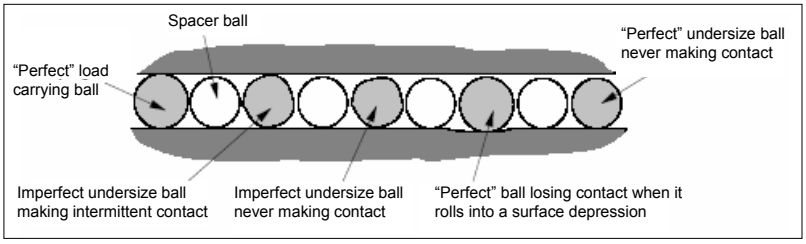
    - At 1000 km, the load capacity is $0.46F_{100km}$!

Design Considerations

- For modular bearings, design data is available from manufacturers
- Sensitive to crashes
- For custom designed bearings, Hertz contact stress theory can be used
  - Hertz contact stress theory is readily implemented in spreadsheet form
  - All rollers are not preloaded evenly and many more rollers than theoretically required may be needed

Accuracy

- Axial: 1 - 5 microns depending on the servo system
  - Specially finished systems can have sub-micron accuracy
- Lateral (straightness) : 0.5 - 10 microns depending on the rails and rolling elements
- Rolling elements are not necessarily round and of the same size:



  - Look for noise spikes at D_{ball}, πD_{ball}, and $2\pi D_{ball}$

Source: Alexander Slocum, *Precision Machine Design*

Accuracy (contd.)

- Elastic averaging helps to reduce high frequency straightness errors, but they still exist
- Entrance and exit path profiles for recirculating elements greatly affect smoothness
- Spacer balls reduce skidding, but decrease load capacity and increase price, so they are very rarely used

Preload

- Prevents lost motion upon load reversal
- If an un-preloaded rolling element is separated from the race by a substantial fluid layer:
  - The fluid layer directly between the rolling element and the race is incompressible
  - It is driven into the race like a needle, leaving a conical depression

Preload for Recirculating Linear Ball Bearings

Preload	Characteristics	Operating conditions	Applications
Heavy	Large Hertz contact area, greater friction and damping	Resists vibration and shock. Withstands cantilevered loads and heavy cutting forces	Machining centers, turning centers
Medium	Modest Hertz contact area and damping, less friction	Withstands light vibration and shock and cantilevered loads. Better for higher speeds	Grinding machines, higher speed machining centers
Light	Small Hertz contact area minimizes friction while maintaining ball spacing to minimize friction	No cantilevered loads. Light and precise operation	Coordinate measuring machines, high speed machines, EDM machines
Very light clearance	No defined contact footprint	Machines with large amounts of thermal growth, minimal cost	Welding machines, automatic tool changers, material handling equipment

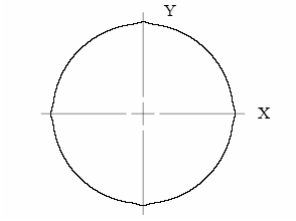
Source:THK Corporation

Characteristics

- Vibration and shock resistance
  - Poor to moderate
  - Significant motion is required periodically to reform a hydrodynamic lubrication layer to prevent fretting
- Damping
  - Additional damping is obtained from the lubrication layer - however the squeeze film area is very small
  - Along the direction of motion, damping is negligible
  - Non-load carrying sliding contact bearings are sometimes added where damping is very important (e.g., grinders)

Friction

- Static friction approximately equals dynamic friction at low speeds, so stick slip is often minimized
- For heavily loaded tables, static friction is still significantly greater than dynamic friction
  - Errors will appear at velocity crossovers:



Source: Alexander Slocum, *Precision Machine Design*

Bearing	"Dimple" size
Sliding contact	10 – 20 μm
Recirculating rolling element	3 – 5 μm
Crossed rollers	1 – 2 μm
Hydrostatic or aerostatic	0 μm

Friction (contd.)

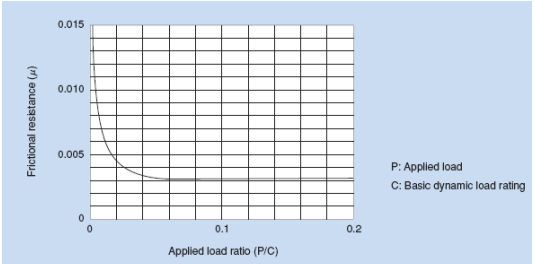


Fig. 8 Relationship between Applied Load Ratio and Frictional Resistance

Table 5 Frictional Resistances (μ) of LM Systems

Types of LM systems	Representative types	Frictional resistance (μ)
LM Guide	SSR, SHS, SNR/SNS, SRS, RSR, HSR, NR/NRS	0.002 to 0.003
	SRG, SRN	0.001 to 0.002
Ball Spline	LBS, LBF, LT, LF	0.002 to 0.003
Linear Bush	LM, LMK, LMF, SC	0.001 to 0.003
LM Stroke	MST, ST	0.0006 to 0.0012
LM Roller	LR, LRA	0.005 to 0.01
Flat Roller	FT, FTW	0.001 to 0.0025
Cross-roller Guide/Cross-roller Table	VR, VRU, VRT	0.001 to 0.0025
Linear Ball Slide	LS	0.0006 to 0.0012
Cam Follower/Roller Follower	CF, NAST	0.0015 to 0.0025

- Friction decreases with increased load until the applied load is roughly 6% of the dynamic load rating
- Higher loads do not increase the friction further.
- Ball bearings have lower friction compared to rollers (0.002 - 0.003 for balls vs. 0.005 - 0.01 for rollers)

Source: THK

Friction (contd.)

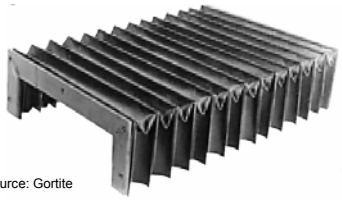
- It is important to use cover plugs on linear guide rail bolt holes
- Seals passing over open holes produce varying friction loads, and can let dirt get into the bearings

Thermal Performance

- Finite friction coefficient generates heat
- Small contact area does not transmits heat well
- Modular bearings themselves may be thermally stable:
  - But can heat from a component (e.g., a spindle) expand the structure and overload the bearings?
- Above 1 m/s, one may want to switch to hydrostatic or aerostatic bearings

Environmental Sensitivity

- Generally intolerant of foreign matter
- Wiper seals are sufficient for low accuracy applications
- For high accuracy applications, bearings should be protected with wipers and/or way covers
- Folded bellows have little to no friction but may fatigue and also easily trap chips on the outside
- Metal sliding covers are smooth on the outside to help deflect chips and coolant but exhibit friction



Source: Gortite

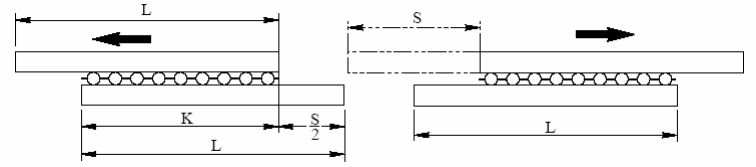
Source: MPC, Milwaukee Protective Covers

Support Equipment

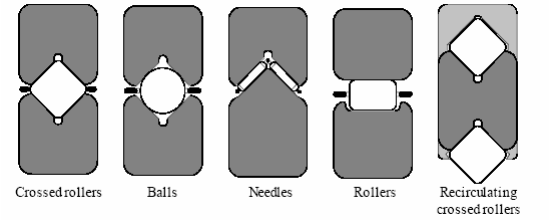
- Many units are sealed for life
- Some units require a periodic application of grease
- For very high cycles (as on a high speed machining center), an oil lubricator should be installed
- Where does the oil go?
  - Design in oil collection gutters into the machine casting
  - This will also facilitate the use of modular hydrostatic bearings (HydroRail™ bearings) that are bolt-for-bolt compatible with rolling element profile rail bearings

Non-Recirculating Crossed Roller Bearings

- Quiet, inexpensive, versatile bearing for short travel
- Rollers travel half the distance of the moving member:



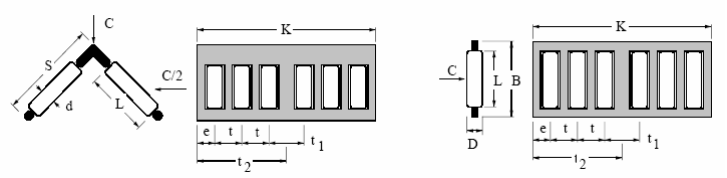
There are many variations on this design:



Source: Alexander Slocum, *Precision Machine Design*

Non-Recirculating Crossed Roller Bearings (contd.)

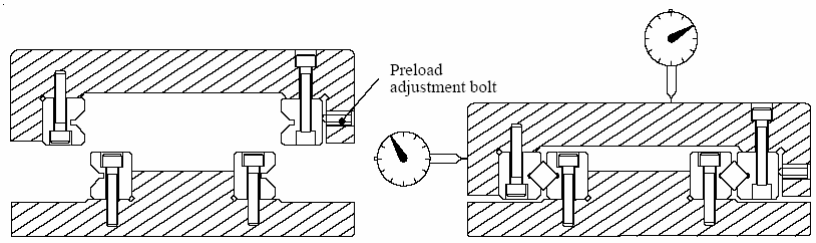
- There are many other variations of non-recirculating linear motion roller bearings
- Typically available modular non-recirculating roller linear bearings (Courtesy of Schneeberger Inc.):



Source: Alexander Slocum, *Precision Machine Design*

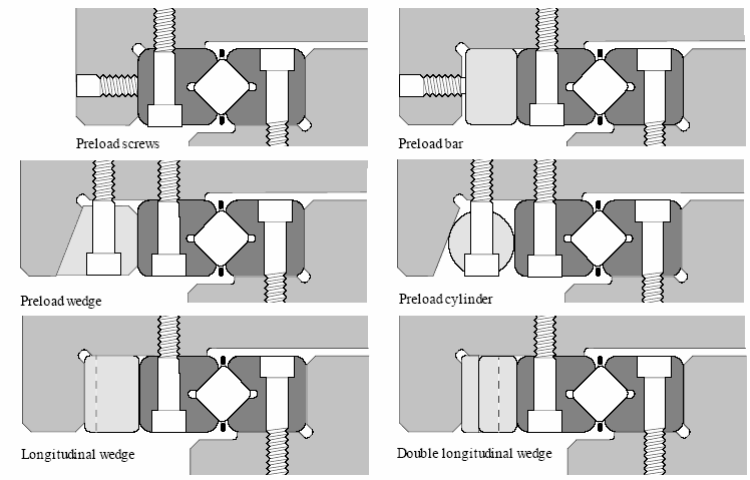
Assembly

- Typical assembly of crossed roller supported slide:



Preload Methods

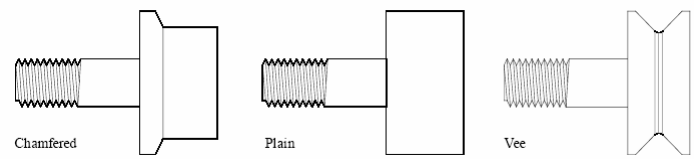
- Methods for preloading crossed roller bearings:



Source: Alexander Slocum, *Precision Machine Design*

Wheels on Rails

- An inexpensive means of obtaining modest performance for a very low cost is to use wheels (cam followers) on rails:

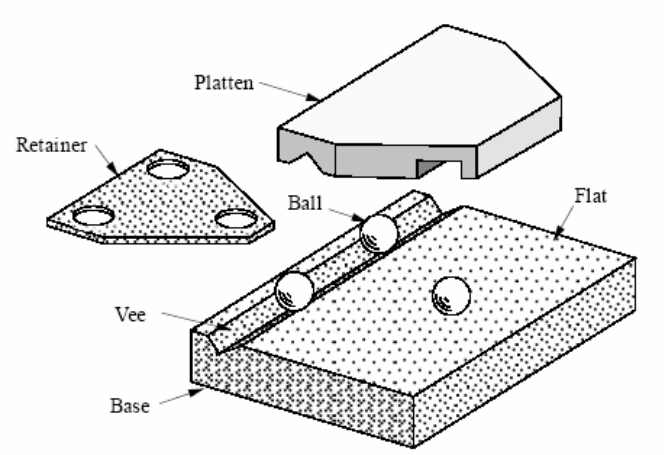


- Kinematic configurations of instrument ball bearings on polished ceramic rails can yield sub-micron performance for a very low cost
- Beware of the formation of frictional polymers on dry-running systems
  - As elements roll, they compress organic molecules in the air onto the surface and build up a layer
  - This layer is not uniform and causes a bumpy ride and velocity control problems

Source: Alexander Slocum, *Precision Machine Design*

Kinematic Systems

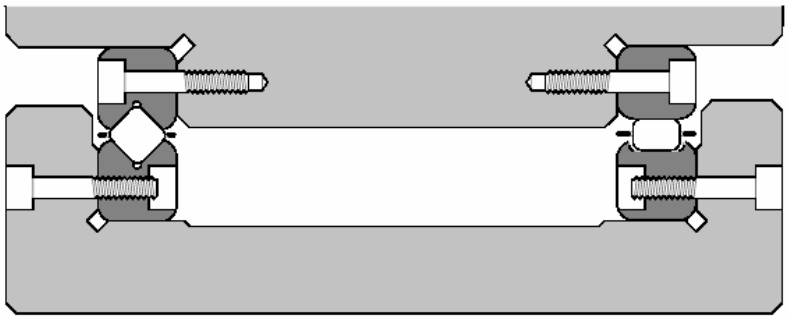
- Kinematic designs are often used:



Source: Alexander Slocum, *Precision Machine Design*

Kinematic Systems (contd.)

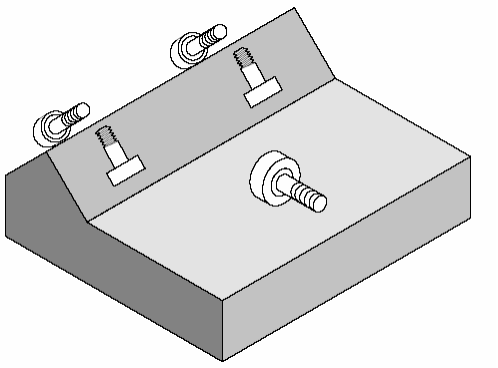
- Quasi-kinematic arrangement of crossed roller bearings and rollers on flat rails:



Source: Alexander Slocum, *Precision Machine Design*

Kinematic Systems

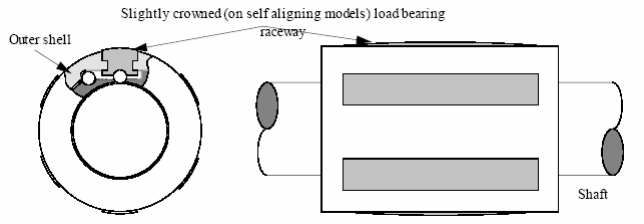
- Concept for a cam-roller based kinematic design (crowned rollers must be used if slip-noise is to be avoided)



Source: Alexander Slocum, *Precision Machine Design*

Ball Bushing Bearings

- Invented in the 1950s by John Thomson
- A linear bearing which incorporates recirculating balls on a round shaft (e.g., a Ball Bushing™ bearing) (Courtesy of Thomson Industries.):



Source: Alexander Slocum, *Precision Machine Design*

Ball Bushing Bearings (contd.)

- Round shafts are inexpensive to grind or hone
- Easy to design and manufacture machines using Ball Bushing™ bearings
- Rotary and torque transmitting designs are available
- Generally intended as a modest accuracy bearing (material handling devices), counterweight guides)

Ball Bushing Bearings (contd.)

- Ball/shaft interface is not optimal for load capacity or stiffness
  - Early machine tools found that by overloading the preload, circular arch grooves cold formed in the shaft
  - Replace the balls and the bearing would be reassembled to perform at higher loads and have greater stiffness
  - This in effect acted as the forerunner of profile rail bearings (linear guides)
- Instrument grades are often used to guide the shafts in gage heads (e.g., an LVDT probe)

Ball Bushing Bearing (contd.)

- Flexures can be used to allow for rail misalignment while allowing a system to be preloaded:

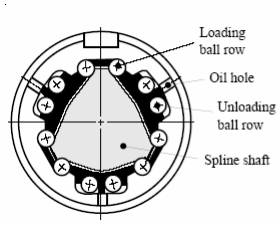
Source: Alexander Slocum, *Precision Machine Design*

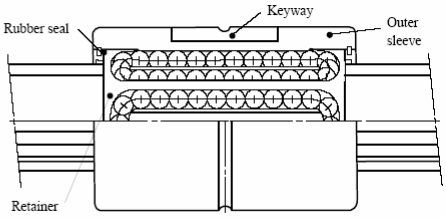
Ball Bushing Bearings (contd.)

- Constant preload is supplied via disc spring washers even if there are variations in rail size
- The flexure connection to the outrigger bearing prevents rail parallelism errors from affecting running friction
- Aluminum extrusions are not too expensive
  - Like the figure above, it is often economical to design a shape with the grooves for the bearing rails to rest in

Ball Splines

- A linear ball bearing on a shaft with circular arch groove spline
- Has increased load capacity and torque transmission capabilities
- Construction of a ball spline for supporting radial and torsional loads (Courtesy of THK Co., LTD.):



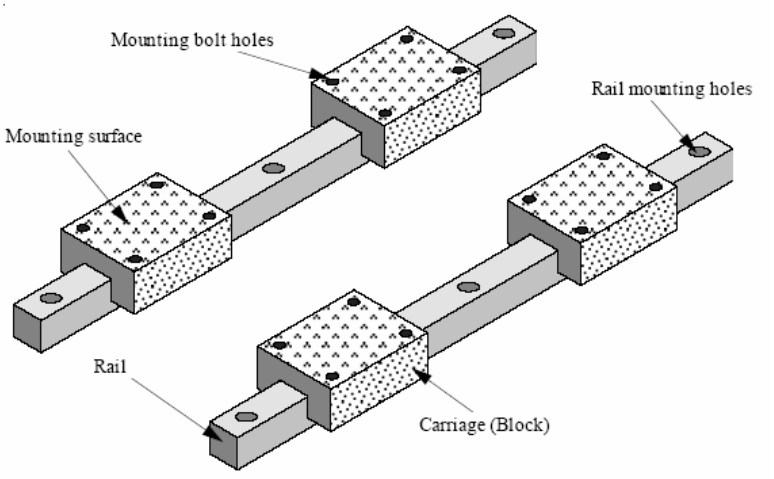


Source: Alexander Slocum, *Precision Machine Design*

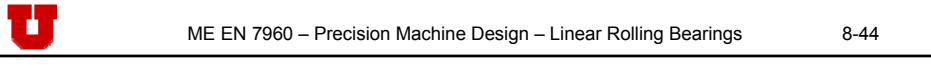
Profile Rail Linear Bearings (Linear Motion Guides)

- With modern grinding techniques, grooved rails can be made very accurate
- Ball/groove interface can be optimized for maximum load capacity and stiffness, or minimum friction
- Easy to design and manufacture machines using linear motion guides
- Analysis of linear guided systems is easily executed using spreadsheets
- Basic components of a linear motion guide bearing system:

Linear Motion Guides

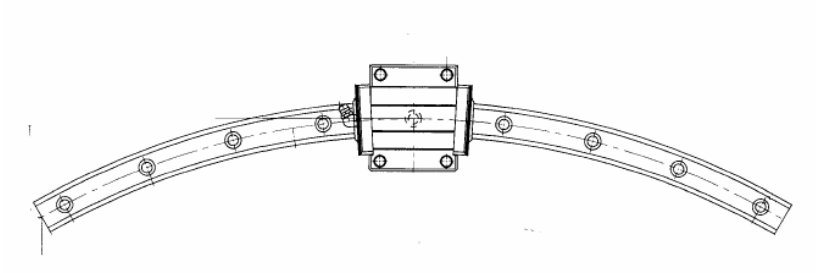


Source: Alexander Slocum, *Precision Machine Design*



Linear Motion Guides

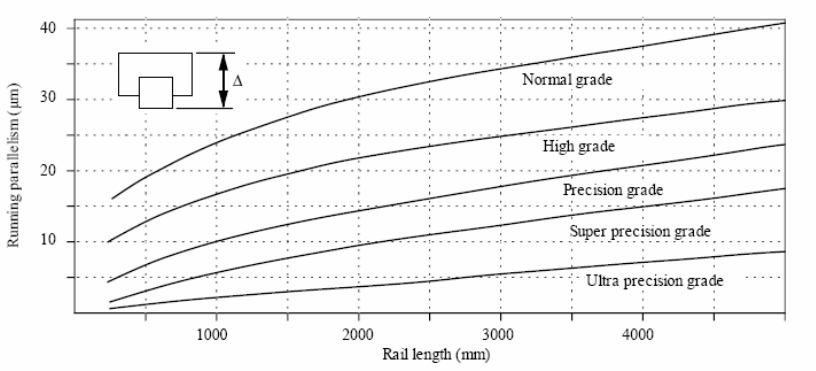
- Specialized Linear Guides are available for curved paths, and also with integral gear racks for long range of motion machines (from THK Corp.)



Source: Alexander Slocum, *Precision Machine Design*

Linear Motion Guide

- Typical running parallelism of linear guides:



- The price ratio of Normal grade and Ultra Precision grade is typically 1:2

Source: Alexander Slocum, *Precision Machine Design*

Modeling Bearings for Selection and Analysis

ME EN 7960 – Precision Machine Design

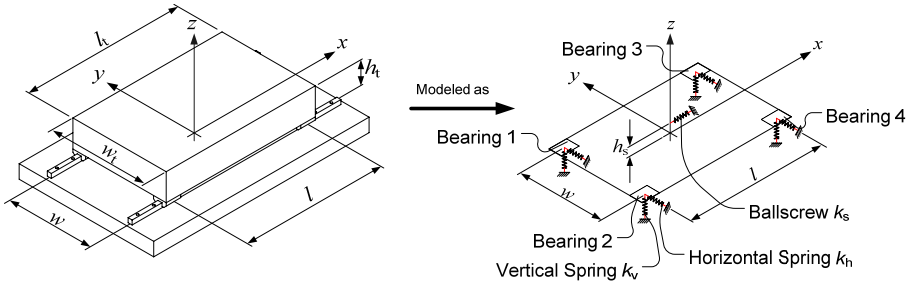
Topic 9

Modeling Principle

- Bearings are complicated machine elements with a large number of rolling elements.
- Not every rolling element makes contact due to manufacturing tolerances.
- Hertzian contact is too difficult and time consuming to model.
- Bearings modeled as steel blocks will exhibit vastly different from real bearings.
- Solution:
  - Rigid body model with bearings modeled as linear springs.
  - Finite element model with bearing modeled as linear springs.
  - Finite element model with bearings modeled as solids having a modulus such that its predicted behavior in finite elements matches that of the real bearing.

Rigid Body Model

- Develop equation of motion



Rigid Body Model

Sum of all kinetic energies:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}I_{xx}\dot{\theta}_x^2 + \frac{1}{2}I_{yy}\dot{\theta}_y^2 + \frac{1}{2}I_{zz}\dot{\theta}_z^2$$

Sum of all potential energies:

$$V = \frac{1}{2}k_{v1}z_1^2 + \frac{1}{2}k_{v2}z_2^2 + \frac{1}{2}k_{v3}z_3^2 + \frac{1}{2}k_{v4}z_4^2 + \frac{1}{2}k_{h1}y_1^2 + \frac{1}{2}k_{h2}y_2^2 + \frac{1}{2}k_{h3}y_3^2 + \frac{1}{2}k_{h4}y_4^2 + \frac{1}{2}k_s x_s^2$$

Describe vertical and horizontal displacements of each spring in terms of the global coordinate system:

$$y_1 = y - \frac{l}{2}\theta_z \qquad z_1 = z + \frac{w}{2}\theta_x + \frac{l}{2}\theta_y$$

$$y_2 = y - \frac{l}{2}\theta_z \qquad z_2 = z - \frac{w}{2}\theta_x + \frac{l}{2}\theta_y$$

$$y_3 = y + \frac{l}{2}\theta_z \qquad z_3 = z + \frac{w}{2}\theta_x - \frac{l}{2}\theta_y$$

$$y_4 = y + \frac{l}{2}\theta_z \qquad z_1 = z - \frac{w}{2}\theta_x - \frac{l}{2}\theta_y \qquad x_s = x + h_s\theta_y$$

Rigid Body Model

$$V = \frac{1}{2}k_v\left[4z^2 + w^2\theta_x^2 + l^2\theta_y^2\right] + \frac{1}{2}k_h\left[4y^2 + l^2\theta_z^2\right] + \frac{1}{2}k_s\left[x^2 + h_s^2\theta_y^2 + 2h_s x\theta_y\right]$$

Use Lagrange:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

Equation of Motion:

$$m\ddot{x} + k_s x + k_s h_s \theta_y = F_x$$
$$m\ddot{y} + 4k_h y = F_y$$
$$m\ddot{z} + 4k_v z = F_z$$
$$I_{xx}\ddot{\theta}_x + k_v w^2 \theta_x = T_x$$
$$I_{yy}\ddot{\theta}_y + k_v l^2 \theta_y + k_s h_s^2 \theta_y + k_s h_s x = T_y$$
$$I_{zz}\ddot{\theta}_z + k_h l^2 \theta_z = T_z$$

Rigid Body Model

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{bmatrix} +$$

$$\begin{bmatrix} k_s & 0 & 0 & 0 & k_s h_s & 0 \\ 0 & 4k_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 & 0 & 0 \\ k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix}$$

Rigid Body Model

Equation of Motion:

$$\vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{Q}$$

Mass Matrix *M*

Acceleration Vector

Stiffness Matrix *K*

Displacement Vector

Load Vector *Q*

Stiffness Matrix *K*:

$$\vec{K} = \begin{bmatrix} k_s & 0 & 0 & 0 & k_s h_s & 0 \\ 0 & 4k_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 & 0 & 0 \\ k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 \end{bmatrix}$$

Rigid Body Model

Quasi-static case: $\ddot{\vec{q}} = \vec{0} \rightarrow \vec{K}\vec{q} = \vec{Q} \rightarrow \vec{q} = \vec{K}^{-1}\vec{Q}$

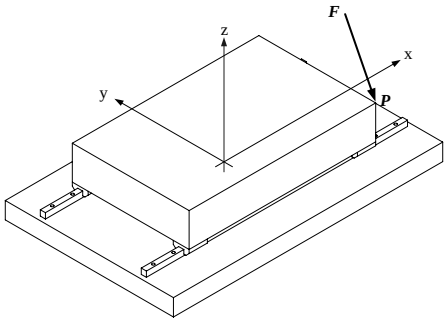
Invert stiffness matrix *K* to obtain compliance matrix *C*:

$$\vec{C} = \begin{bmatrix} \frac{k_v l^2 + k_s h_s^2}{k_v k_s l^2} & 0 & 0 & 0 & \frac{-h_s}{k_v l^2} & 0 \\ 0 & \frac{1}{4k_h} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4k_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k_v w^2} & 0 & 0 \\ \frac{-h_s}{k_v l^2} & 0 & 0 & 0 & \frac{1}{k_v l^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{k_h l^2} \end{bmatrix}$$

Rigid Body Model

Create load vector:

$$\vec{T} = \vec{P} \times \vec{F} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$



Calculate displacement vector:

$$\vec{q} = \vec{C}\vec{Q} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix} \begin{matrix} \vec{q}_{lin} \\ \\ \\ \vec{q}_{rot} \\ \\ \end{matrix}$$

$$\Delta\vec{P} = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{P} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix} \times \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \end{bmatrix}$$

Rigid Body Model

Stiffness at Point *P*:

$$k_x = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_x|} \quad k_y = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_y|} \quad k_z = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_z|}$$

Bearing Displacements

Determine bearing displacements:

Bearing 1: $$\Delta\vec{B}_1 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_1 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{1,x}\\ B_{1,y}\\ B_{1,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{1,x}\\ \Delta B_{1,y}\\ \Delta B_{1,z}\end{bmatrix}$$

Bearing 2: $$\Delta\vec{B}_2 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_2 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{2,x}\\ B_{2,y}\\ B_{2,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{2,x}\\ \Delta B_{2,y}\\ \Delta B_{2,z}\end{bmatrix}$$

Bearing 3: $$\Delta\vec{B}_3 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_3 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{3,x}\\ B_{3,y}\\ B_{3,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{3,x}\\ \Delta B_{3,y}\\ \Delta B_{3,z}\end{bmatrix}$$

Bearing 4: $$\Delta\vec{B}_4 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_4 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{4,x}\\ B_{4,y}\\ B_{4,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{4,x}\\ \Delta B_{4,y}\\ \Delta B_{4,z}\end{bmatrix}$$

Bearing Loads

Horizontal (=lateral) force

$$F_{B1,y} = F_{B1,h} = k_h \Delta B_{1,y}$$
$$F_{B2,y} = F_{B2,h} = k_h \Delta B_{2,y}$$
$$F_{B3,y} = F_{B3,h} = k_h \Delta B_{3,y}$$
$$F_{B4,y} = F_{B4,h} = k_h \Delta B_{4,y}$$

Vertical force

$$F_{B1,z} = F_{B1,v} = k_v \Delta B_{1,z}$$
$$F_{B2,z} = F_{B2,v} = k_v \Delta B_{2,z}$$
$$F_{B3,z} = F_{B3,v} = k_v \Delta B_{3,z}$$
$$F_{B4,z} = F_{B4,v} = k_h \Delta B_{4,z}$$

Check for brinelling:

Horizontal force of bearing *i* must not exceed the static load rating in the lateral direction:

$$\left|F_{Bi,h}\right| \le \frac{C_{o,l}}{f_w}$$

$C_{o,l}$ = static load rating in lateral direction
f_w = application factor

Bearing Loads

Positive vertical force of bearing *i* must not exceed the static load rating in the reverse radial direction:

$$F_{Bi,h+} \le \frac{C_{o,rr}}{f_w}$$

$C_{o,rr}$ = static load rating in reverse radial direction

Negative vertical force of bearing *i* must not exceed the static load rating in the reverse radial direction:

$$\left|F_{Bi,h-}\right| \le \frac{C_o}{f_w}$$

C_o = static load rating in radial direction

Equivalent dynamic load rating determines bearing life:

$$P_e = X\left|F_{Bi,v}\right| + Y\left|F_{Bi,h}\right|$$

Where *X* and *Y* are factors provided by the manufacturer that reflect the load inside the bearing as a result of a vertical and lateral load

Bearing Life

$$L = 100 \cdot \left(\frac{C_d}{f_w P_e}\right)^a$$

L = bearing life in km
C_d = dynamic load rating
f_w = application factor
a = exponent (a=3 for balls, 10/3 for rollers)

Resonance Frequencies

Equation of motion: $\vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{Q}$

For free vibration, solve homogenous solution: $\vec{Q} = \vec{0} \quad \rightarrow \vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{0}$

Assume harmonic solution: $u = U\cos(\omega t)$

$$\frac{du}{dt} = \dot{u} = -U\omega\sin(\omega t)$$

$$\frac{du^2}{dt^2} = \ddot{u} = -U\omega^2\cos(\omega t) \quad \rightarrow \ddot{u} = -u\omega^2$$

Then: $\vec{M}\vec{q}\,\omega^2 + \vec{K}\vec{q} = \vec{0}$ and $\vec{M}\vec{q}\,\omega^2 + \vec{K}\vec{q} = \vec{0}$

Resonance Frequencies

$$\begin{bmatrix} k_s - m\omega^2 & 0 & 0 & 0 & -k_s h_s & 0 \\ 0 & 4k_h - m\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v - m\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 - I_x\omega^2 & 0 & 0 \\ -k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 - I_y\omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 - I_z\omega^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The non-trivial solutions of this eigenvalue problem are found by calculating the determinate:

$$\det\left|K - M\omega^2\right| = 0$$

Here:

$$\left(k_h l^2 - I_z\omega^2\right)\left(k_v w^2 - m\omega^2\right)\left(4k_v - m\omega^2\right)\left(4k_h - m\omega^2\right)\left(I_y m\omega^4 - \left(k_s I_y + k_v l^2 m + k_s h_s^2 m\right)\omega^2 + k_s k_v l^2\right) = 0$$

Resonance Frequencies

Then:

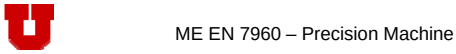
$$\omega_a = \sqrt{\frac{k_h l^2}{I_z}}$$

$$\omega_b = \sqrt{\frac{k_v w^2}{I_x}}$$

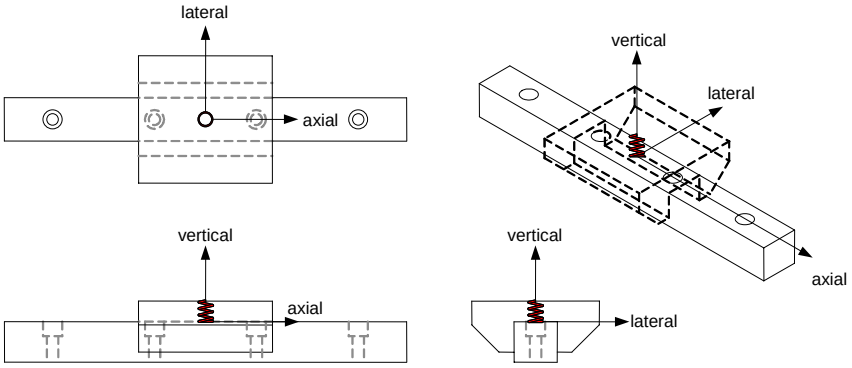
$$\omega_c = \sqrt{\frac{4k_v}{m}}$$

$$\omega_d = \sqrt{\frac{4k_h}{m}}$$

$$\omega_{e,f} = \sqrt{\frac{1}{2I_Y m}\left[k_s I_y + \left(k_v l^2 + k_s h_s^2\right)m \pm \sqrt{k_s^2\left(I_y^2 + h_s^4 m^2 + 2h_s^2 I_y m\right) + k_v^2 l^4 m^2 + 2k_s k_v l^2 m\left(h_s^2 m - I_y\right)}\right]}$$



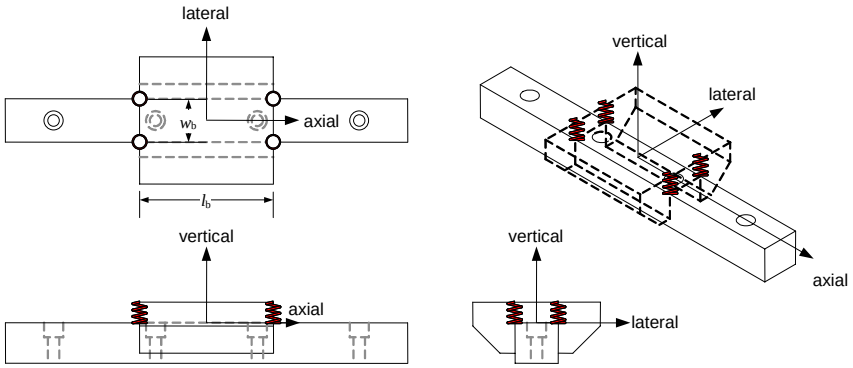
Using Springs – Single Springs



If no moment stiffness is known, use a single linear spring located in the center of the bearing. The spring connects the top surface of the rail with the top surface of the bearing.

$$k_{axial} = 0 \quad k_{lateral} = k_{lateral,literature} \quad k_{vertical} = k_{vertical,literature}$$

Using Springs – Multiple Springs

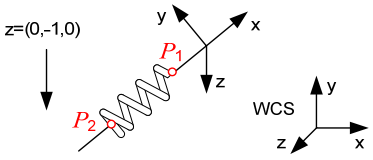


Use for better quality analysis and if stress concentrations are bothersome.

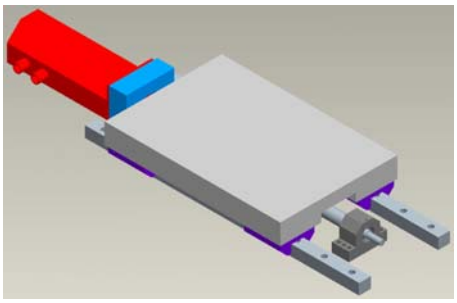
$$k_{axial} = 0 \quad k_{lateral} = \frac{k_{lateral,literature}}{4} \quad k_{vertical} = \frac{k_{vertical,literature}}{4}$$

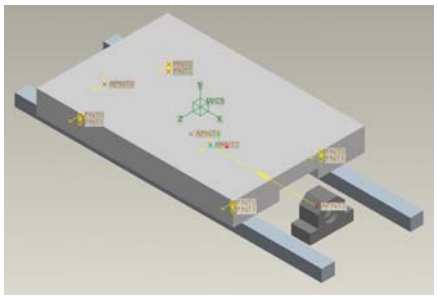
Modeling Springs in Pro/MECHANICA

- Bearings between axes are modeled as point-to-point springs.
- Point-to-point springs require two datum points (P_1, P_2) that define the length and position of the spring.
- The vector between the two datum points defines the springs' x-axis.
- The z-axis of the spring is defined as a vector relative to the World Coordinate System (WCS).
- Springs that attach to solids require the rotational DOF's of the datum points to be constrained.



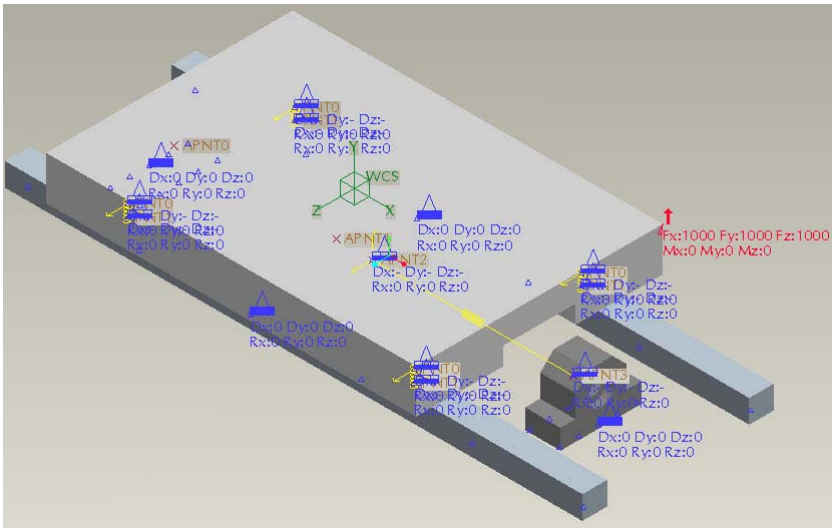
Linear Axis with 4 Bearings and Ball Screw



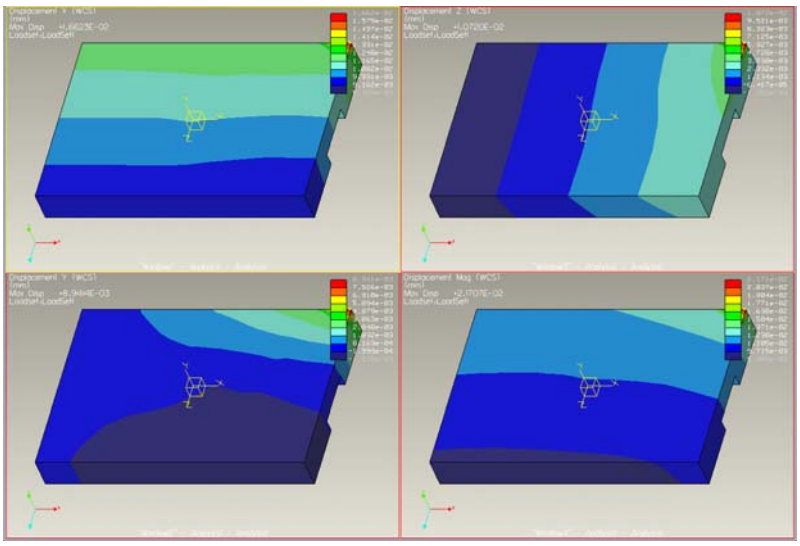


Suppress non-relevant components (motor, motor mount).
Suppress non-relevant features (bolt holes).
Replace bearings and ball screw with springs.

Linear Axis with Springs and Constraints



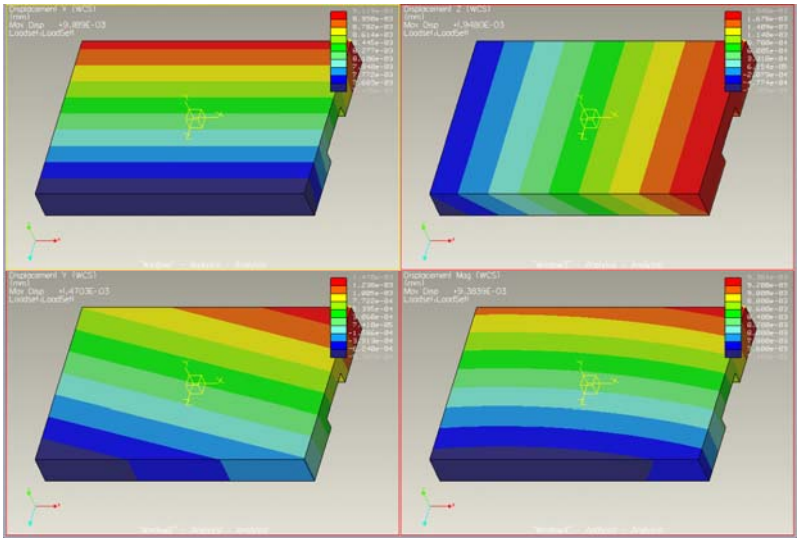
Linear Axis with Springs and Steel



Comparison

	Rigid Body Model	FEA with Springs and Steel Table	
Disp in x [μm]	8.07	16.6	+106%
Disp in y [μm]	1.91	10.7	+460%
Disp in z [μm]	1.45	8.94	+517%

Linear Axis with Springs and Stiff-Stuff



Comparison

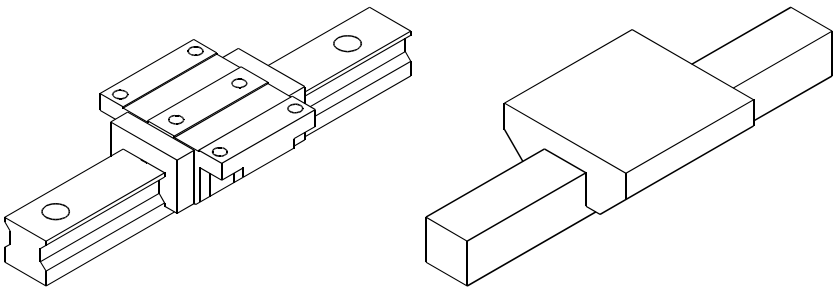
	Rigid Body Model	FEA with Springs and Steel Table		FEA with Springs and Steel from Stiff-Stuff	
Disp in x [μm]	8.07	16.6	+106%	9.12	+13%
Disp in y [μm]	1.91	10.7	+460%	1.95	+2.1%
Disp in z [μm]	1.45	8.94	+517%	1.47	+1.3%

Modeling Bearings as Solids

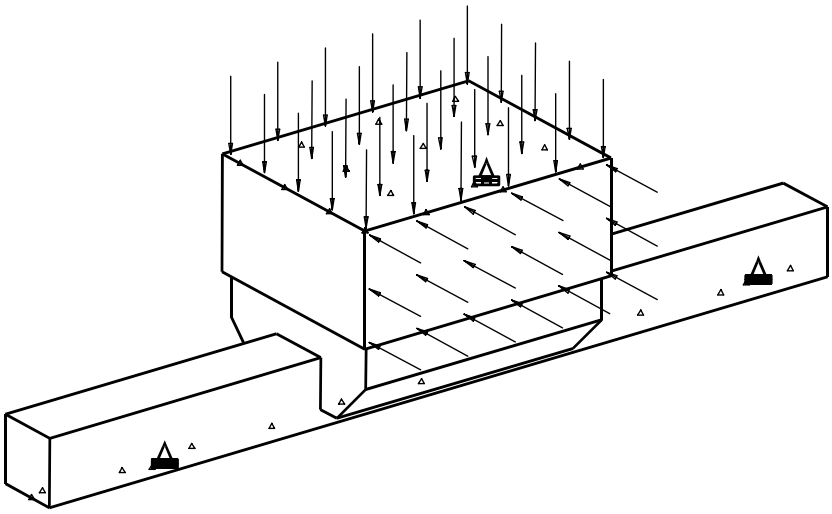
- Modeling bearings as solids has the advantage of having a model that looks realistic.
- Avoids the replacing of the bearings with springs (tedious and error-prone).
- BUT: Analysis time will significantly increase and convergence might be an issue.

Technique

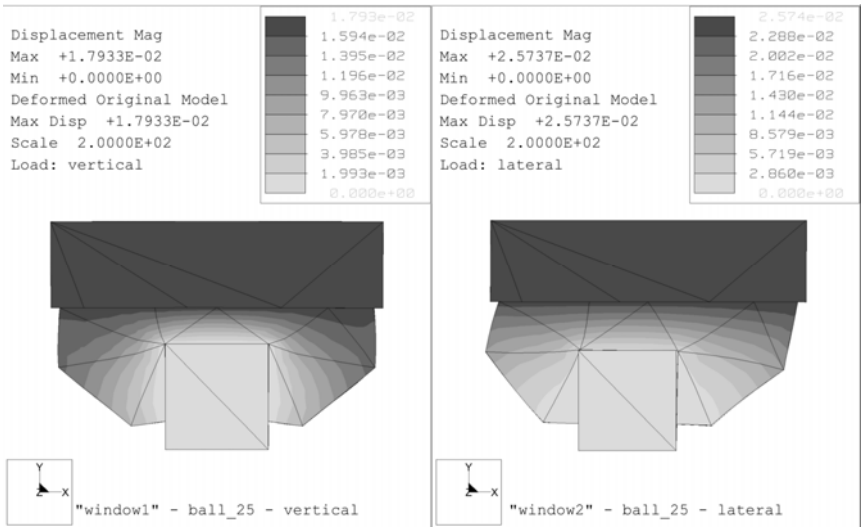
- Identify a Young's modulus that will allow the solid bearing to deform just as the real bearing would under the same loading.
- Remove unnecessary details that will increase analysis time.



Define FEA Analysis that Mimics a Vertical and Lateral Load



FEA Results



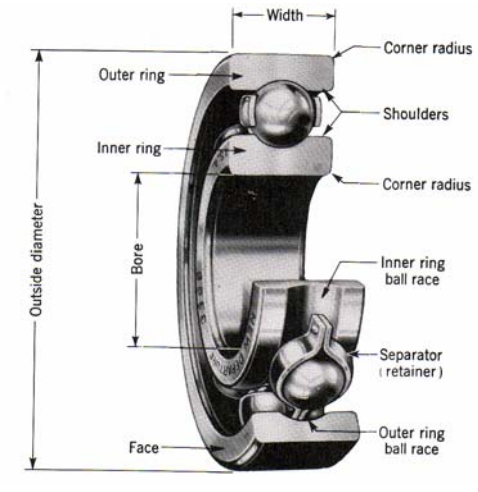
Create a Modulus Table

Type	Size	Preload	Length [mm]	Width [mm]	Height [mm]	$E_{vertical}$ [N/mm^2]	$E_{lateral}$ [N/mm^2]
Ball	25	8%	81	70	29.5	3650	2040
Ball	35	8%	105	100	40	3560	2100
Ball	45	8%	133	120	50	3690	2180
Ball	55	8%	159	140	57	3505	2370
Roller	25	13%	91	70	30	5400	2920
Roller	35	13%	114	100	41	7060	3650
Roller	45	13%	140	120	51	8500	4060
Roller	55	13%	166.5	140	58	8800	4180

Rotary Rolling Contact Bearings

ME EN 7960 – Precision Machine Design
Topic 10

Ball Bearing Nomenclature



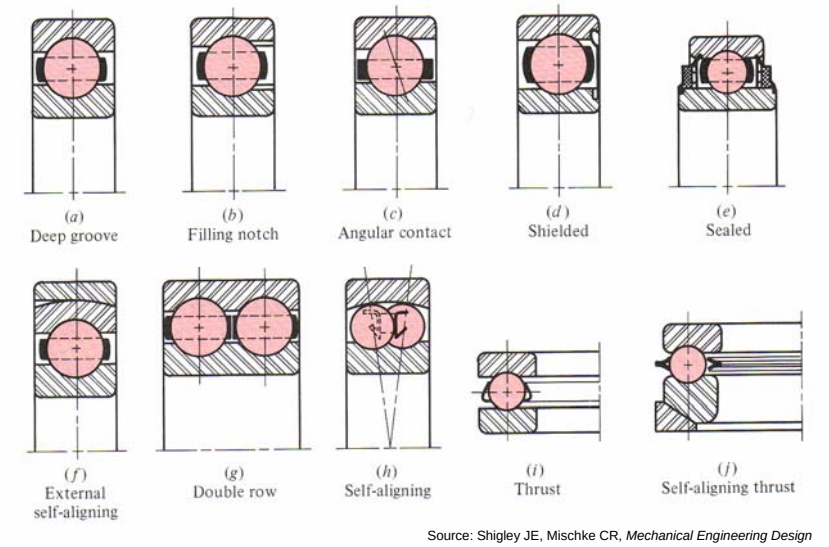
Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Rotary ball bearings are defined through geometry and performance.

Geometry:
- Outside diameter
- Inside diameter
- Width

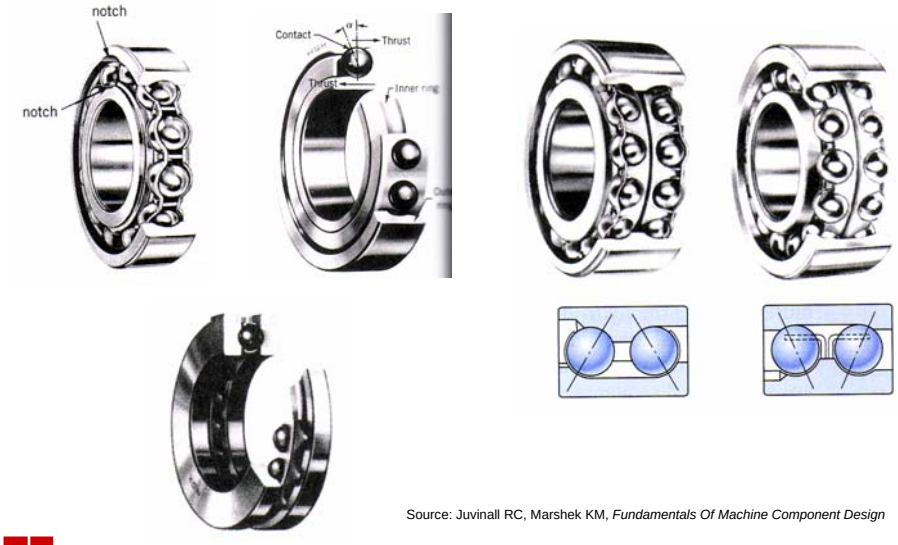
Performance:
- Load capacity (radial, axial, moment)
- Stiffness (radial, axial, moment)
- Runout
- Bearing life
- Allowable speed
- Lubrication
- etc.

Ball Bearing Types (Rotary)



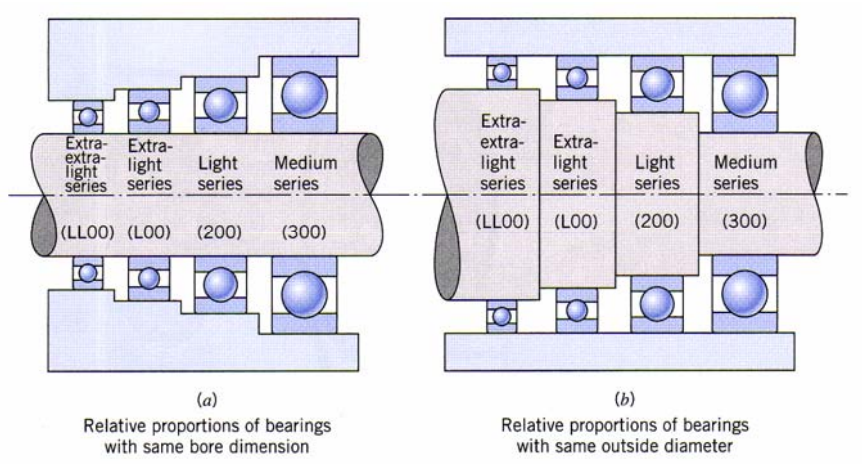
Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Ball Bearings (Rotary)



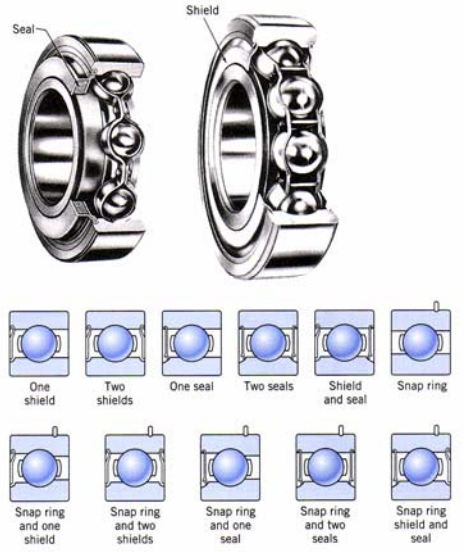
Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Bearing Sizes



Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Bearing Seals



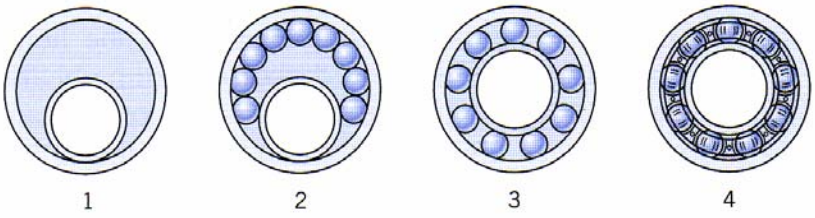
Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Ball Bearings – Details

- The ball has a smaller radius than the groove
- The load capacity of a bearing is limited by the contact stresses between the rolling elements and the races

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

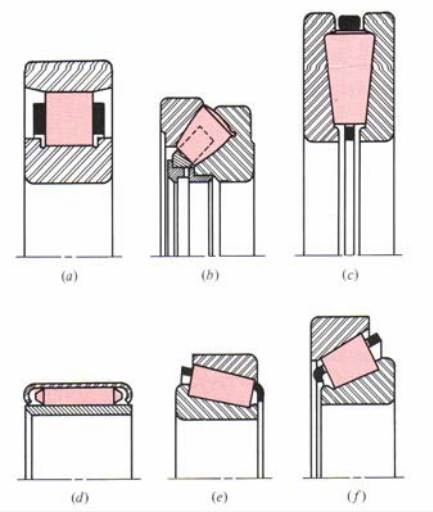
Ball Bearings - Assembly



Step 1: Move the inner race to one side of the out race
Step 2: Locate the largest gap between the two races and insert the balls in the groove
Step 3: Distribute the balls evenly, thereby centering the inner race with respect to the outer race
Step 4: Place the separator (retainer)

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Roller Bearings (Rotary)



a) Straight roller
b) Spherical roller
c) Tapered roller thrust
d) Needle
e) Tapered roller
f) Steep-angle tapered.

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Roller Bearings (Rotary)

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Ball vs. Roller Bearings

- Roller bearings are stiffer and have a higher load capacity than comparably sized ball bearings
  - This is due to the type of contact, line contact for rollers vs. point contact for balls
- Ball bearings have a lower friction
  - This also is a function of contact type.\
- Ball bearings can often be operated at higher speeds
- Most ball bearings can take modest axial loads for “free”
  - Only tapered rollers can take axial loads
- Ball bearings are less expensive than roller bearings

Bearing Characteristics

Bearing Type	Radial Capacity	Thrust Capacity	Limiting Speed	Radial Stiffness	Axial Stiffness
Deep-groove ball	Moderate	Moderate – both directions	High	Moderate	Low
Maximum-capacity ball	Moderate (plus)	Moderate – one direction	High	Moderate (plus)	Low (plus)
Angular contact ball	Moderate	Moderate (plus) – one direction	High (minus)	Moderate	Moderate
Cylindrical roller	High	None	Moderate (plus)	High	None
Spherical roller	High	Moderate – both directions	Moderate	High (minus)	Moderate
Needle roller	Moderate to high	None	Moderate to very high	Moderate to high	None
Single-row tapered	High (minus)	Moderate (plus) – one direction	Moderate	High (minus)	Moderate
Double-row tapered	High	Moderate – both direction	Moderate	High	Moderate
Four-row tapered	High (plus)	High – both direction	Moderate (minus)	High (plus)	High
Ball thrust	None	High – one direction	Moderate (minus)	None	High
Roller thrust	None	High (plus) – one direction	Low	None	High (plus)
Tapered roller thrust	Locational only	High (plus) – one direction	Low	None	High (plus)

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Failure Modes – Surface Fatigue

- Surface fatigue is the dominant failure mode
- The cyclic subsurface Hertzian shear stresses produced by the curved surfaces in rolling contact may initiate and propagate cracks that ultimately dislodge particles and generate surface pits
- Typically, the raceways pit first, resulting in noise, vibration, and heat

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Failure Modes – Brinelling

- Static loads on stationary bearings may cause brinelling of the races
- The resulting local discontinuities cause vibration, noise, and heat

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Selection Criteria

- When selecting bearings, both failure modes need to be considered. As such, we need to examine:
  - Resistance to surface fatigue, expressed as *basic dynamic load rating* C_d
  - Resistance to brinelling, expressed as *basic static load rating* C_s

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Basic Load Ratings

- Basic load ratings are a standardized measure provided by the bearing industry to quantify the rolling element bearing's ability to resist surface fatigue and brinelling
- The basic static load rating C_s is a measure of the resistance to failure by brinelling
- The basic dynamic load rating C_d is a measure of resistance to failure due to surface fatigue

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

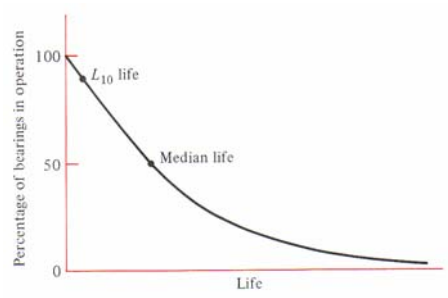
Bearing Life

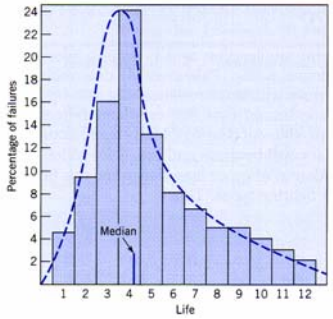
Two identical bearings tested under different loads P_1 and P_2 will have respective lives L_1 and L_2 according to:

$$\frac{L_1}{L_2} = \left(\frac{P_1}{P_2} \right)^a \quad (1)$$

Where:
a = 3 for ball bearings
a = 10/3 for roller bearings

Experiments with identical bearings under identical conditions reveal:





Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Example 1: Timken Bearing

A Timken ball bearing is rated as follows:
At a rated load of 2140 lb, the bearing has a life of 3000 hr at 500 rpm

Solution: Timken Bearing

A Timken ball bearing is rated as follows:
At a rated load of 2140 lb, the bearing has a life of 3000 hr at 500 rpm.

a) Calculate the bearing life *L*

$$L = tn = 3000\ hr \cdot 60 \frac{\min}{hr} \cdot 500 \frac{rev}{\min} = 90 \cdot 10^6\ rev$$

b) Determine the basic dynamic load rating

From (3):

$$\frac{L}{10^6} = \left(\frac{C_d}{P}\right)^3 \quad \rightarrow \quad C_d = \left(\frac{L}{10^6}\right)^{\frac{1}{3}} P = \left(\frac{90 \cdot 10^6}{10^6}\right)^{\frac{1}{3}} \cdot 2140\ lb = 8263\ lb$$

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Basic Dynamic Load Rating

The basic dynamic load rating C_d is defined to be the largest stationary radial load that 90 percent of a group of apparently identical bearings will survive for 1 million revolutions (inner race rotating, outer race fixed) with no evidence of failure by surface fatigue

The bearing life *L* for a given load *P* can be determined as follows:

$$L = \left(\frac{C_d}{P}\right)^a 10^6$$

L = bearing life (revolutions to failure)
C_d = basic dynamic load rating (90% reliability)
P = applied bearing load
a = exponent (*a* = 3 for ball bearings and a = 10/3 for roller bearings)

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Reliability Adjustments

Reliability adjustment factors, based on actual failure rate data, allows a designer to select bearings for reliabilities higher than 90%

$$L_R = K_R L \qquad (4)$$

L_R = reliability-adjusted bearing life
K_R = reliability life-adjustment factor
L = bearing life

Reliability Life-Adjustment Factor K_R for Bearing Reliabilities Different from R = 90%		
Reliability R (%)	**Probability of Failure P (%)**	**K_R**
50	50	5.0
90	10	1.0
95	5	0.62
96	4	0.53
97	3	0.44
98	2	0.33
99	1	0.21

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Adjustments for Impact Loads

Rated load ratings are basic on static, continuous loading. Time-varying loads reduce the bearing life and need to be considered when selecting bearings

The applied equivalent dynamic load P_e is modified by the estimated *impact factor* I_F:

$$P = I_F \cdot P_e \qquad (5)$$

Estimated Impact Factors for Various Applications	
Type of Application	**Impact Factor I_F**
Uniform load, no impact	1.0 – 1.2
Precision gearing	1.1 – 1.2
Commercial gearing	1.1 – 1.3
Toothed belts	1.1 – 1.3
Light impact	1.2 – 1.5
V-Belts	1.2 – 2.5
Moderate Impact	1.5 – 2.0
Flat belts	1.5 – 4.5
Heavy impact	2.0 – 5.0

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Selection Procedure

1. From free-body diagram, determine the *radial load* F_r and *axial thrust load* F_a
2. Determine the *design life requirement* L_d for the bearing
3. Determine the *reliability* R appropriate to the application and select the corresponding *life-adjustment factor* K_R
4. Assess the severity of any shock or impact associated with the application and select an appropriate *Impact Factor* I_F
5. Calculate the dynamic equivalent radial load P_e

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Selection Procedure (contd.)

$$P_{e1} = X_{d1}F_r + Y_{d1}F_a$$
$$P_{e2} = X_{d2}F_r + Y_{d2}F_a \quad (6)$$

Where: X_d = dynamic radial load factor
Y_d = dynamic axial (thrust) factor

If $P_{e1} > P_{e2}$ then $P_e = P_{e1}$, otherwise $P_e = P_{e2}$

Approximate Radial Load Factors for Selected Bearing Types

Bearing Type	Dynamic				Static			
	X_{d1}	Y_{d1}	X_{d2}	Y_{d2}	X_{s1}	Y_{s1}	X_{s2}	Y_{s2}
Single-row radial ball bearing	1	0	0.55	1.45	1	0	0.6	0.5
Single-row angular contact bearing (shallow angle)	1	0	0.45	1.2	1	0	0.5	0.45
Single-row angular contact bearing (steep angle)	1	0	0.4	0.75	1	0	0.5	0.35
Double row radial ball bearing	1	0	0.55	1.45	1	0	0.6	0.5
Double-row angular contact bearing (shallow angle)	1	1.55	0.7	1.9	1	0	1	0.9
Double-row angular contact bearing (steep angle)	1	0.75	0.6	1.25	1	0	1	0.65
Straight roller bearing	1	0	-	-	1	0	1	0

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Bearing Selection Procedure (contd.)

6. Calculate the basic dynamic load rating requirement

$$C_d = \left(\frac{L}{K_R \cdot 10^6} \right)^{\frac{1}{a}} I_F P_e \qquad (7)$$

C_d = required dynamic load rating to give a bearing reliability of R percent
L = life (revolutions)
K_R = reliability adjustment factor
I_F = application impact factor
P_e = equivalent radial load
a = exponent equal to 3 for ball bearings or 10/3 for roller bearings

7. With C_d enter a basic load rating table and find the smallest bearing with a load rating of at least C_d

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Example 2: Rolling Element Bearing Selection

A support shaft for a new product has been designed with a diameter of 1.6 inches. The force analysis shows that:

Radial bearing load F_r = 370 lb
Axial bearing load F_a = 130 lb
Shaft speed n = 350 rpm
Design life specification is 10 years of operation, 50 days/year, 20hr/day
Design reliability specification is R = 95%
The shaft is V-belt driven

Solution: Rolling Element Bearing Selection

A support shaft for a new product has been designed with a diameter of 1.6 inches. The force analysis shows that:

1. Find design life in revolutions:

$$L = tn = 10\,yr \cdot 50\frac{days}{yr} \cdot 20\frac{hr}{day} \cdot 60\frac{\min}{hr} \cdot 350\frac{rev}{\min} = 2.1 \cdot 10^8\ rev$$

2. Find life-adjustment factor K_R for R = 95%

$$K_R = 0.62$$

3. Find impact factor I_F for application. Here, the shaft is V-belt driven.

$$I_F = 1.9$$

4. Calculate equivalent dynamic bearing load. Assume a single-row, deep-groove ball bearing.

Solution: Rolling Element Bearing Selection (contd.)

From table for deep-groove bearing: $X_{d1} = 1$, $Y_{d1} = 0$, $X_{d2} = 0.55$, $Y_{d2} = 1.45$

$$P_{e1} = X_{d1}F_r + Y_{d1}F_a = 1.0 \cdot 370 + 0.0 \cdot 130 = 370\ lb$$
$$P_{e2} = X_{d2}F_r + Y_{d2}F_a = 0.55 \cdot 370 + 1.45 \cdot 130 = 392\ lb$$
$$\rightarrow P_e = 392\ lb$$

5. Calculate basic dynamic load requirement for R = 95%

$$C_d = \left(\frac{L}{K_R \cdot 10^6}\right)^{\frac{1}{3}} I_F P_e = \left(\frac{2.1 \cdot 10^8}{0.62 \cdot 10^6}\right)^{\frac{1}{3}} \cdot 1.9 \cdot 392 = 5192\ lb$$

Consulting manufacturer's tables, we need to find a deep-groove bearing with a basic dynamic load rating of at least 5192 lb, a bore diameter of at least 1.6 inches, and an operational speed of at least 350 rpm.

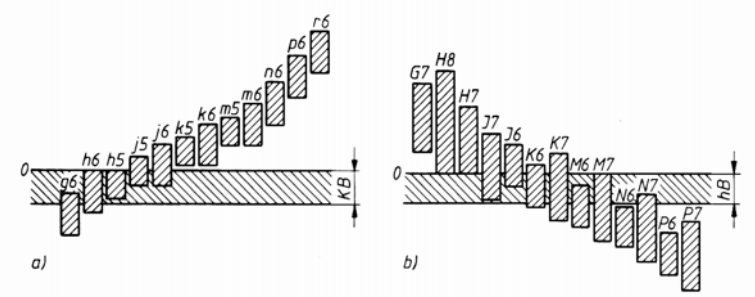
Bearing Quality

- Bearing quality (= tolerances) has been standardized by the Annular Bearing Engineers' Committee (ABEC)
- There are four primary grades of precision: ABEC 1, 5, 7, and 9
- ABEC 1 is the standard grade suited for most normal applications
  - For bearing bores between 35 and 50mm, tolerances are +0.0000 in. to -0.0005 in
- ABEC 9 is the highest grade for applications of extreme accuracy
  - For bearing bores between 35 and 50mm, tolerances are +0.00000 in. to -0.00010 in

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Bearing Fits

- Fits should be chosen according to manufacturer's recommendation. Alternatively, use the following rule:
  - The *rotating ring* should have an interference fit with its mating member (to ensure that the bearing ring will not slip or rotate)
  - The *non-rotating ring* should have a close push fit (to allow axial slip in case of thermal expansion)



K_b = tolerance for bearing inside diameter of inner race
h_b = tolerance for bearing outside diameter of outer race

Source: Matek W, Muhs D, Wittel H, *Roloff/Matek Maschinenelemente*

Typically Bearing Life Requirement

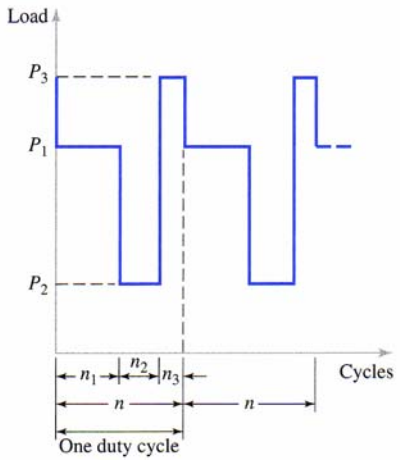
Typical Bearing Life Requirements	
Type of Application	**Life [hr]**
Instruments and apparatus for infrequent use	Up to 500
Aircraft engines	500 – 2,000
Machines for short or intermittent operation where service interruption is of minor importance	4,000 – 8,000
Machines for intermittent service where reliable operation is of great importance	8,000 – 14,000
Machines for 8 hr service which are not always fully utilized	14,000 – 20,000
Machines for continuous 24 hr service	50,000 – 60,000
Machines for continuous 24 hr service where reliability is of extreme importance	100,000 – 200,000

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Spectrum Loading

If a bearing is subjected to a spectrum of different applied loads during each duty cycle in the operation of a machine, two basic methods are available:

1. Assume that the largest load is applied to the bearing at every revolution, even though the actual load may be smaller during some segments of the operation
   - Bearing is oversized (not economical but safe)
2. Consider load spectrum
   - Requires thorough knowledge about spectrum



Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Spectrum Loading (contd.)

From bearing life equation:

$$[C_d]^a \cdot 10^6 = L_i P_i^a = L_1 P_1^a = L_2 P_2^a = \cdots$$

Where L_i = bearing life at a given load P_i

For a duty cycle as shown in the right figure, the total number of cycles is n, of which n_1 occur at a load P_1, and n_2 cycles occur at a load P_2, etc.

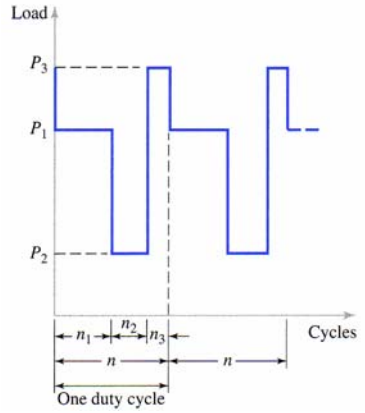
$$n_1 + n_2 + \cdots + n_i = n$$

$$\alpha_1 = \frac{n_1}{n} \equiv \textit{fraction of cycles at } P_1$$

$$\alpha_2 = \frac{n_2}{n} \equiv \textit{fraction of cycles at } P_2$$

$$\vdots$$

$$\alpha_i = \frac{n_i}{n} \equiv \textit{fraction of cycles at } P_i$$



Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Spectrum Loading (contd.)

Where: L_1 = cycles to failure if all cycles were loaded at P_1
L_2 = cycles to failure if all cycles were loaded at P_2
L_i = cycles to failure if all cycles were loaded at P_i

$$\alpha_i L_d = \textit{total cycles at } P_i$$

Palmgren-Miner linear damage rule, failure can be expected if:

$$\sum \frac{\alpha_i L}{L_i} = 1 \qquad (8)$$

With:

$$L_i = \frac{[C_d]^a \cdot 10^6}{P_i^a} \qquad (9)$$

(9) in (8):

$$\sum \frac{(\alpha_i L_d) P_i^a}{[C_d]^a \cdot 10^6} = 1 \qquad (10)$$

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Spectrum Loading (contd.)

Or:
$$\alpha_1 P_1^a + \alpha_2 P_2^a + \cdots + \alpha_i P_i^a = \frac{[C_d]^a \cdot 10^6}{L_d} \qquad (11)$$

Or:
$$C_d = \left[\frac{L}{K_R \cdot 10^6}\right]^{\frac{1}{a}} \left[\sum \alpha_i \left((I_F P_e)_i\right)^a\right]^{\frac{1}{a}} \qquad (12)$$

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Example: Spectrum Loading

The load analysis of a newly proposed rotating shaft has been found as shown in the table below.
The total design life is to be 10^7 revolutions with a reliability of 97%.
A single-row deep groove bearing is preferred.

a) Find the basic dynamic load rating required for this application based on the load spectrum

b) Find the basic dynamic load rating required for this application based on the maximum load

Duty Cycles For Application			
Variable	**Segment 1**	**Segment 2**	**Segment 3**
Radial load F_r [lb]	2000	1000	5000
Axial load F_a [lb]	500	900	0
Shock loading I_F	Light shock	Moderate shock	Moderate shock
Cycles n_i/duty cycle [number of cycles]	1000	2000	100
Operational speed N_{op} [rpm]	3600	7200	900

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Solution: Spectrum Loading

a) Find the basic dynamic load rating required for this application based on the load spectrum

For a reliability of 97%, the *life-adjustment factor* K_R = 0.44

The *impact factor* I_F for light shock is chosen as I_F = 1.4, and for medium shock I_F = 1.8

Total number cycles: $n = n_1 + n_2 + n_3 = 1000 + 2000 + 100 = 3100$

Cycle fractions: $\alpha_i = \frac{n_i}{n} \rightarrow \alpha_1 = \frac{n_1}{n} = \frac{1000}{3100} = 0.32$

Similarly for α_2 and α_3: $\alpha_2 = \frac{n_2}{n} = \frac{2000}{3100} = 0.65$ *and* $\alpha_3 = \frac{n_3}{n} = \frac{100}{3100} = 0.03$

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Solution: Spectrum Loading (contd.)

For a deep groove bearing:

$$X_{d1} = 1 \quad Y_{d1} = 0 \quad X_{d2} = 0.55 \quad Y_{d2} = 1.45$$

Determine equivalent dynamic bearing loads P_{e1} and P_{e2}:

$$P_{e1} = X_{d1} F_r + Y_{d1} F_a$$
$$P_{e2} = X_{d2} F_r + Y_{d2} F_a$$

	Segment 1	**Segment 2**	**Segment 3**
P_{e1} [lb]	2000	1000	5000
P_{e2} [lb]	1825	3450	2750
P_e [lb]	2000	3450	5000

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

Solution: Spectrum Loading (contd.)

Using (12):

$$C_d = \left[\frac{L}{K_R \cdot 10^6}\right]^{\frac{1}{a}} \left[\sum \alpha_i \left((I_F P_e)_i\right)^a\right]^{\frac{1}{a}}$$

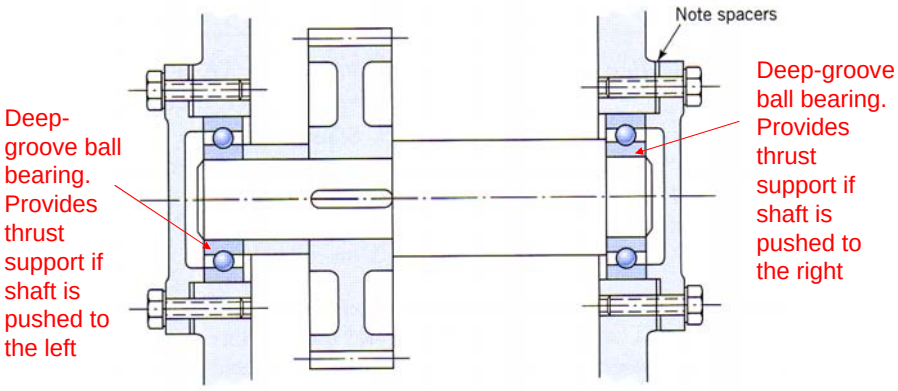
$$C_d = \left[\frac{10^7}{0.44 \cdot 10^6}\right]^{\frac{1}{3}} \sqrt[3]{\left(0.32[1.4 \cdot 2000]^3 + 0.65[1.8 \cdot 3450]^3 + 0.03[1.8 \cdot 5000]^3\right)} = 15{,}460\ lb$$

b) Find the basic dynamic load rating required for this application based on the maximum load.

$$C_d = \left[\frac{10^7}{0.44 \cdot 10^6}\right]^{\frac{1}{3}} \cdot 1.8 \cdot 5000 = 25{,}470\ lb$$

Source: Collins JA, *Mechanical Design of Machine Elements and Machines*

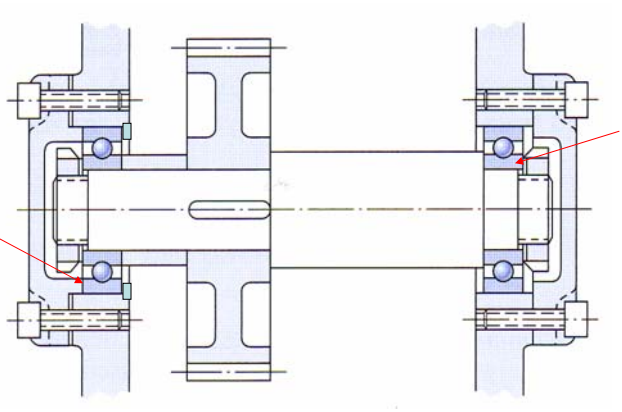
Bearing Mount for Thrust



WARNING!!!!!
Thermal expansion of the shaft creates thrust force, possibly overloading the bearings.

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Bearing Mount for Thrust (contd.)



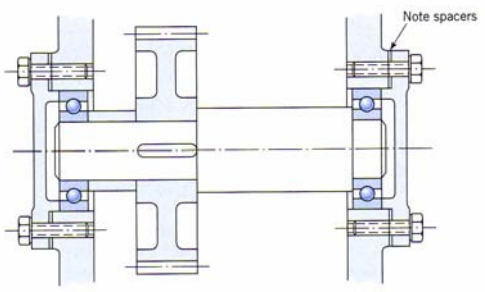
RECOMMENDED!!!!
The thrust force is provided by a single bearing. The second bearing only takes radial loads because it is allowed to slide axially.

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Example: Thrust from Thermal Expansion

A shaft with a diameter of 40 mm and a length of 200 mm is rigidly supported at both ends with a single-row deep-groove bearing (#6008). The bearings have a rated basic dynamic load rating of $C_{d(90)}$ = 16.8 kN. The gear is creating a radial load of 5000 N while the thrust force is negligible. The shaft spins at 350 rpm.

a) Assuming that radial load on the left bearing is 3000 N, what is the bearing life for 98% reliability?
b) Assuming that the shaft heats up to 5°C above room temperature, what is the expected bearing life? (Assume the housing to be rigid and at room temperature).



Solution: Thrust from Thermal Expansion

a) Assuming that radial load on the left bearing is 3000 N, what is the bearing life for 98% reliability?

For a deep groove bearing:

$$X_{d1}=1 \quad Y_{d1}=0 \quad X_{d2}=0.55 \quad Y_{d2}=1.45$$

Determine equivalent dynamic bearing loads P_{e1} and P_{e2}:

$$P_{e1}=X_{d1}F_r+Y_{d1}F_a$$
$$P_{e2}=X_{d2}F_r+Y_{d2}F_a$$

Here: $P_e=3000N$

Solution: Thrust from Thermal Expansion (contd.)

Bearing Equation: $$C_d=\left(\frac{L}{K_R\cdot 10^6}\right)^{\frac{1}{a}} I_F P_e$$

Solve for bearing life L:

$$L=\left(\frac{C_d}{I_F\cdot P_e}\right)^3\cdot K_R\cdot 10^6=\left(\frac{16800}{1.2\cdot 3000}\right)^3\cdot 0.33\cdot 10^6=3.35\cdot 10^7\,rev$$

Or in terms of operating time:

$$t=\frac{L}{n}=\frac{3.35\cdot 10^7\,rev}{350\frac{rev}{\min}\cdot 60\frac{\min}{hr}}=1{,}600hr$$

Solution: Thrust from Thermal Expansion (contd.)

b) Assuming that the shaft heats up to 5°C above room temperature, what is the expected bearing life? (Assume the housing to be rigid and at room temperature.

Thermal expansion coefficient for steel: $\alpha = 12 \cdot 10^{-6} \frac{1}{K}$

Linear expansion: $\Delta l = \alpha \cdot \Delta T \cdot L = 12 \cdot 10^{-6} \cdot 5 \cdot 0.2 = 12 \cdot 10^{-6} m = 12 \mu m$

How much force is created if the shaft is prevented to expand 12μm?

For a bar, the stiffness is given as: $k = \frac{AE}{L}$

In general: $k = \frac{F}{\delta}$

Solution: Thrust from Thermal Expansion (contd.)

Combine:

$$F = \frac{AE \cdot \Delta L}{L} = \frac{\pi d^2 E \cdot \Delta L}{4L} = \frac{\pi (0.04m)^2 \cdot 207 \cdot 10^9 \frac{N}{m^2} \cdot 12 \cdot 10^{-6} m}{4 \cdot 0.2m} = 15607N$$

For a deep groove bearing:

$$X_{d1} = 1 \quad Y_{d1} = 0 \quad X_{d2} = 0.55 \quad Y_{d2} = 1.45$$

Determine equivalent dynamic bearing loads P_{e1} and P_{e2}:

$$P_{e1} = X_{d1} F_r + Y_{d1} F_a = 3000N$$
$$P_{e2} = X_{d2} F_r + Y_{d2} F_a = 24280N$$

Here:

$$\rightarrow \quad P_e = 24280N$$

Example: Thrust from Thermal Expansion (contd.)

Solve for bearing life *L*:

$$L = \left(\frac{C_d}{I_F \cdot P_e}\right)^3 \cdot K_R \cdot 10^6 = \left(\frac{16800}{1.2 \cdot 24280}\right)^3 \cdot 0.33 \cdot 10^6 = 6.32 \cdot 10^4 rev$$

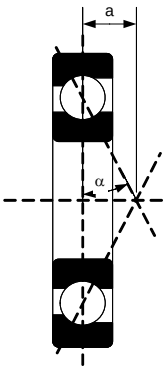
Or in terms of operating time:

$$t = \frac{L}{n} = \frac{6.23 \cdot 10^4 rev}{350 \frac{rev}{\min} \cdot 60 \frac{\min}{hr}} = 3.0 hr$$

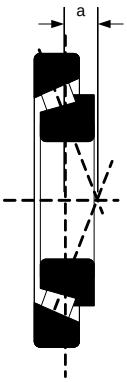
The thermal expansion reduces the bearing life by a factor of 532!!!!

Angular Contact and Tapered Roller Bearings

- All tapered bearings including angular contact ball bearings transmit loads from the inner to the outer ring at an angle with respect to the center line
- This angle creates an offset *a* from the center of the bearing

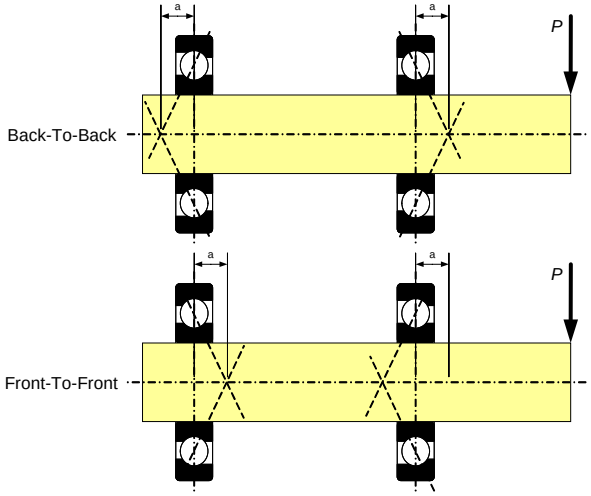


Angular Contact Bearing



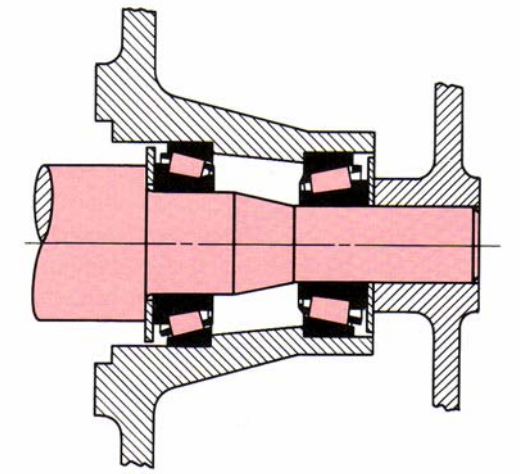
Tapered Roller Bearing

Two possible mounts: Back-To-Back (DB) and Front-To-Front (DF)



Back-To-Back mounting increases the effective spacing between the bearings, thereby reducing the bearing loads required to counteract the applied moment
Front-to-Front has reduced moment stiffness compared to Back-to-Back. For the case where the load lines intersect at the same point, moment stiffness is zero and the bearing is "self-aligning".

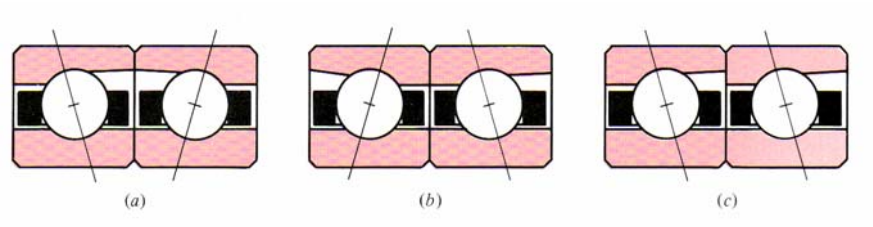
Back-To-Back Examples



Spindle of Front Loader Washing Machine

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

Angular Contact Bearing Arrangements



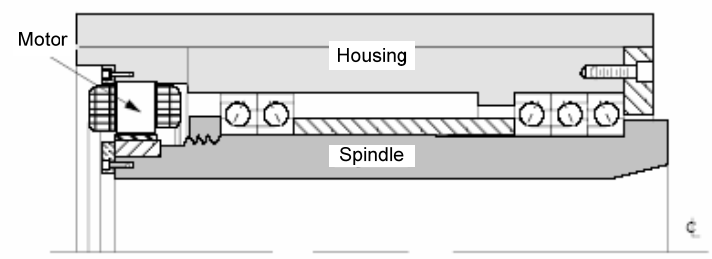
a) Front-To-Front (DF) mount. Reduced moment stiffness results in "self-alignment"
   - The system is thermally unstable for a rotating inner race
   - Differential radial and axial growth both tend to increase the preload

b) Back-To-Back (DB) mount. Increased moment stiffness results in rigid bearing support
   - For a fixed outer race, the shaft expands axially and radially more than the housing
   - The preload remains relatively constant

c) Parallel (DT) mount

Source: Shigley JE, Mischke CR, *Mechanical Engineering Design*

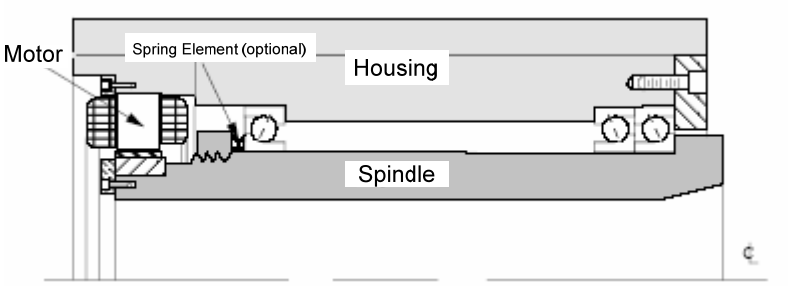
Mounting Strategies for Spindles
Fixed-Supported



Source: Alexander Slocum, *Precision Machine Design*

- This is the most common, least expensive, and least desirable method
- It can lead to decreased radial stiffness and accuracy if the fit is too loose
- Early bearing failure if the fit is too tight
- Consider using a surface treatment in the bore to decrease friction

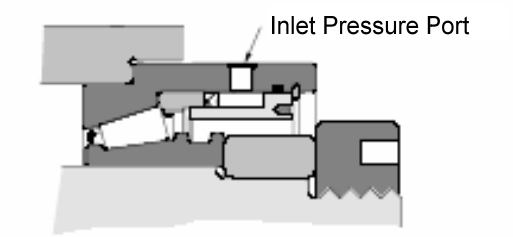
Mounting Strategies for Spindles
Fixed-Thermocentric



Source: Alexander Slocum, *Precision Machine Design*

- The radial and axial spacing are set so thermal growths cancel each other
- Optionally, a spring element is used to preload the system to allow for deviations from ideal growth
- This is a very difficult configuration to design and thus is not often used for a multi-purpose spindle

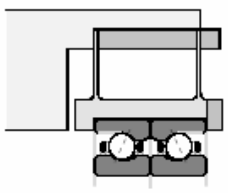
Mounting Strategies for Spindles - Fixed Preload



Source: Alexander Slocum, *Precision Machine Design*

- Use a hydraulic device to maintain a fixed preload on the bearings (Hydra-Rib™)
- This is an effective, but more expensive, method

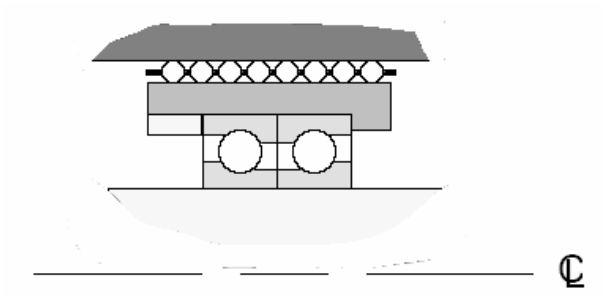
Mounting Strategies for Spindles
Fixed-Diaphragm Flexure



Source: Alexander Slocum, *Precision Machine Design*

- Use a diaphragm flexure to support the bearing set which must have freedom to move axially
  - This is a moderate cost method that does require extra room and thus is not often used
- Method for bearing mounting that provides:
  - Good radial stiffness
  - Low axial stiffness
  - Accommodation of axial thermal growth of the supported shaft

Mounting Strategies for Spindles
Fixed-Rolling Element

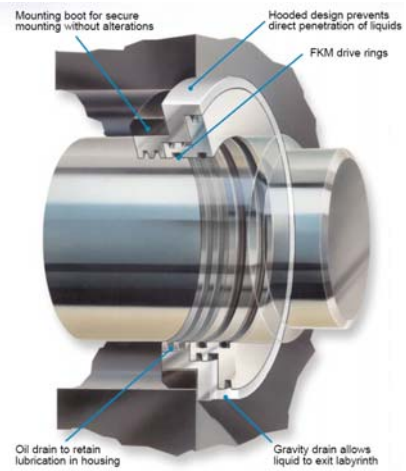


Source: Alexander Slocum, *Precision Machine Design*

- Use a die-set bushing as a micro-motion flexural rolling element bearing

Seals

- Rotary motion rolling element bearings are relatively easy to seal using labyrinth, wiper, or screw-thread type seals
- At high speeds, non-contact seals (Hydra Seals) are required to avoid heat generation
  - Even better sealing can be achieved with positive air pressure
- High pressure coolant bouncing off of a part can force its way past non-contact

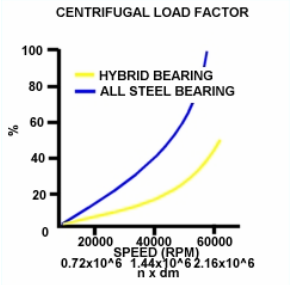


Accuracy

- Total axial and radial error motions can be as low as ¼ µm but typically are on the order of ½ - 1 µm
- Commercially available spindles, are available with total error motions (TIR/2) of 1/4 - 1/2 µm
- Angular contact bearings typically have three times the speed capability of radial contact bearings
- Angular contact bearings are generally the most often used type of ball bearing in precision machinery
- For high-speed spindles, ceramic balls are well suited

Ceramic Ball Bearings

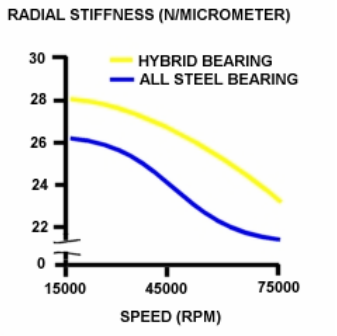
- 40% as dense as steel, the resulting reduction in weight reduces centrifugal forces imparted on the rings, reducing skidding, allowing 30 to 50% higher running speeds with less lubrication
- Ceramic balls have a smoother finish than steel, vibration and spindle deflection is reduced allowing higher speeds

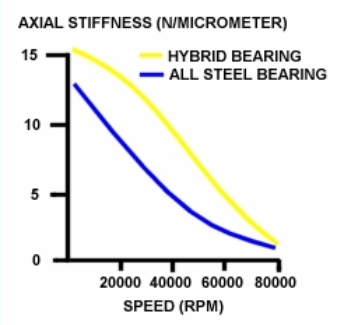


Source: Bearing Works

Ceramic Ball Bearings (contd.)

- Silicon nitride balls have a 50 % higher modulus of elasticity than steel, which means a 15 to 20% increase in rigidity, improving accuracy

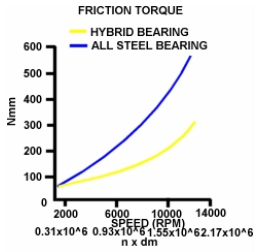


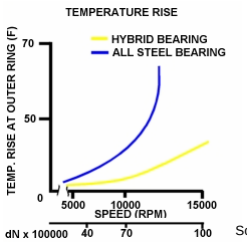


Source: Bearing Works

Ceramic Ball Bearings (contd.)

- A low thermal expansion coefficient allows HYBRIDS with ceramic balls to undergo smaller changes in contact angle reducing preload variations improving life and maintaining tolerances
- Ceramic Hybrid Ball Bearings truly are "anti-friction"
- Lower friction leads to less wear, less lubrication, less energy consumption, reduced sound level and extends life

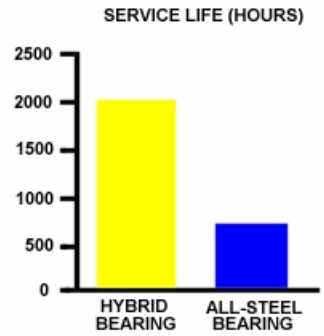




Source: Bearing Works

Ceramic Ball Bearings (contd.)

- With their numerous advantages ceramic hybrid ball bearings typically yield 5 to 10 times longer life than conventional steel-steel ball bearings in most applications



Source: Bearing Works

Externally Pressurized Fluid Film Bearings

ME EN 7960 – Precision Machine Design
Topic 11

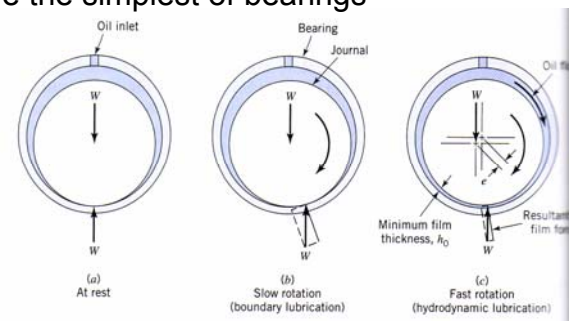
General Characteristics

- Bearings with an externally pressurized fluid film have no mechanical contact
  - Instead, bearing surfaces are separated by a thin fluid layer
- As a result, error motions are extremely small and the system is extremely well damped (harmonics die quickly)

Source: Alexander Slocum, *Precision Machine Design*

Hydrodynamic Bearings

- Viscous fluids are dragged between bodies as they move past each other, so the rotating shaft acts like a pump
- The pressure gradient is limited, and so is the load capacity and stiffness
- They are the simplest of bearings



Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Hydrodynamic Bearings

- Optimum speed is reached immediately after shaft has lifted off
  - Increasing speed further causes significant losses due to viscous friction
- Generally not useful for design that require a wide range of rotational speeds (e.g. spindles)
- Very useful for applications that require a single, constant speed (e.g. power generators)

Source: Juvinall RC, Marshek KM, *Fundamentals Of Machine Component Design*

Hydrostatic and Aerostatic Bearings

- Metered flow to each side of the bearing creates a pressure differential proportional to the displacement
- Load capacity and stiffness can be very high
- They require the expense of a clean pressure supply system

Overview

	Liquid (Oil, Water)	**Gas (Air, Nitrogen)**
Type	Capillary, orifice, slot or diaphragm restricted	Porous, orifice, or slot restricted
Typical applications	Large machining forces (Milling machines, etc.)	Moderate forces (Measuring equipment, grinding spindles, diamond turning)
Load capacity	Very high	Moderate
Stiffness	Very high	High
Damping	Very high	Moderate
Friction	Low at low speeds	Very low at all speeds

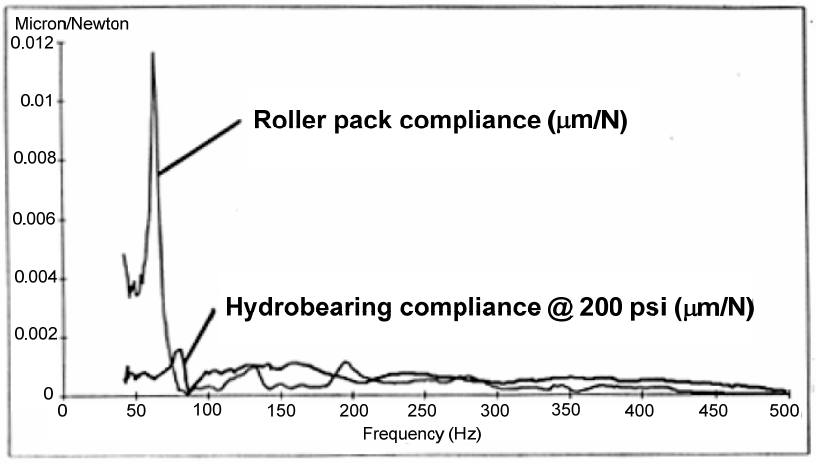
Performance

- Accuracy
  - Axial: Limited only by control system
  - Lateral: Limited by straightness of rails and isolation from pressure system
- Applied loads
  - Large surface area allows for high load capacity
  - Virtually insensitive to crashes
- Preload
  - Most designs are inherently preloaded
- Stiffness
  - Easily made many times greater than other components in the machine
  - Dynamic stiffness is very high due to squeeze film damping

Performance (contd.)

- Vibration and shock resistance
  - Excellent for liquid bearings
  - Modest-to-poor for gas bearings
- Damping capability
  - Excellent normal to direction of motion, due to squeeze film damping
  - Low along direction of motion
  - Bearing area, gap, and stiffness must be considered to maximize squeeze film damping
  - Squeeze film damping greatly affects the dynamic stiffness

Performance (contd.)



Source: Alexander Slocum, *Precision Machine Design*

Performance (contd.)

- Friction
  - Zero static friction
  - Dynamic friction depends on gap and fluid viscosity
- Thermal performance
  - Finite dynamic friction coefficient generates heat
  - Fluid flowing at pressure released to atmospheric pressure shears and generates heat equal to pump power
  - A cooler is often needed to control fluid temperature
  - Expanding gas creates cooling (Joule Thompson cooling)

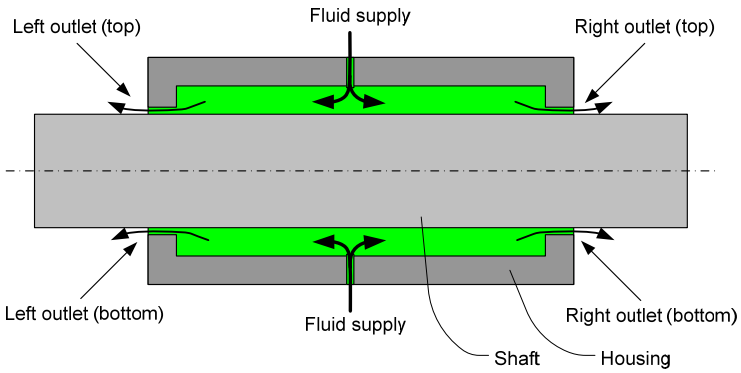
Performance (contd.)

- Environmental sensitivity
  - Very intolerant of dirt
  - Where the fluid has to flow past a tiny gap (a capillary or an orifice), it can clog
  - Gear pump flow-dividers and self-compensating bearings are "self cleaning“
  - A particle lodged in a small gap can score the bearing or the rail

Support Equipment

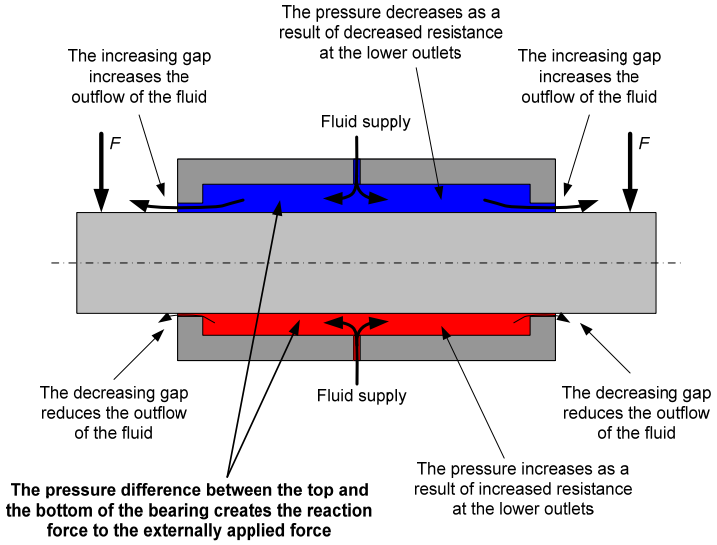
- Pumps
  - Screw-type pumps are most quiet
  - Piston pumps are noisiest
  - Accumulators and pressure relief valves are needed to keep pressure pulsations from increasing error motions
  - Pumps generate heat
  - If linear motors are used on the machine, they will require an order-of-magnitude more cooling than the hydrostatic bearings
- Filters
  - Air bearings ideally are fitted with desiccant dryers
  - Fluid bearings require filters
  - Water bearings require fluid chemistry control
  - Centrifugal filters work well, but are expensive
  - Cartridge filters must be changed

Theory of Fixed Compensation

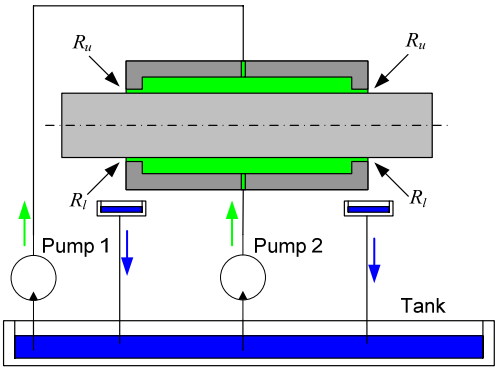


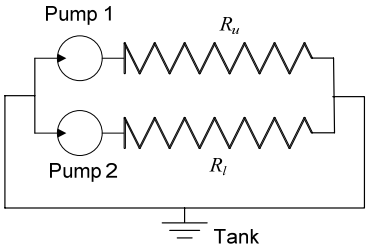
- Fluid flow into the bearing is regulated by a resistance that is created at the outlets of the bearing
  - outlets are formed by a small gap between the housing and the shaft

Theory of Fixed Compensation



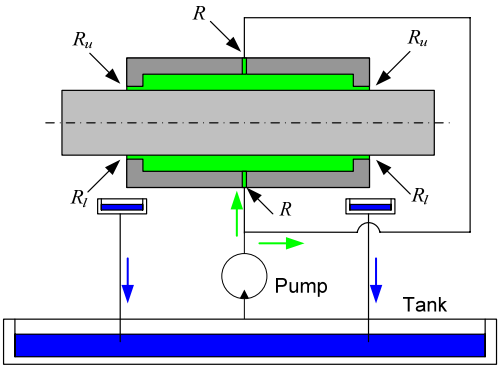
Theory of Fixed Compensation

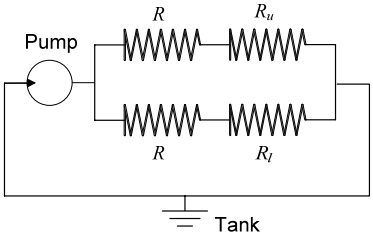




- If a bearing only regulates the outflow of the fluid, a separate pump for each side of the bearing is required to allow the pressure to be regulated independently

Theory of Fixed Compensation





- To eliminate the need for a separate pump for each bearing side, a resistance *R* at the inlets is added

Theory of Fixed Compensation (contd.)

- The difference in pressure between the upper and lower pads of the bearing is:

$$\Delta p = p_u - p_l = p_s\left(\frac{R_u}{R+R_u} - \frac{R_l}{R+R_l}\right)$$

- For a nominal gap *h* and small excursions δ of the structure:

$$R_u = \frac{\gamma}{(h-\delta)^3} \qquad R_l = \frac{\gamma}{(h+\delta)^3}$$

γ = constant of proportionality

- The difference in pressure across the bearing is:

$$\Delta p = p_s\gamma\left[\frac{1}{R(h-\delta)^3+\gamma} - \frac{1}{R(h+\delta)^3+\gamma}\right]$$

Theory of Fixed Compensation (contd.)

- If the inlet flow resistance *R* was zero, the bearing could support no load
- If the inlet flow resistance was infinite, the bearing could support no load
  - There must be some ideal inlet resistance (compensation) between these two extremes
- Taking the partial derivative of the pressure difference with respect to the inlet flow resistance;
  - Ignoring all terms with δ^2 and higher terms:

$$\frac{\partial \Delta p}{\partial R} = p_s\gamma h^2\left[\frac{-(h-3\delta)}{\left(Rh^2(h-3\delta)+\gamma\right)^2} - \frac{(h+3\delta)}{\left(Rh^2(h+3\delta)+\gamma\right)^2}\right]$$

Theory of Fixed Compensation (contd.)

- The "optimal" inlet flow resistance to maximize load capacity is:

$$R = \frac{\gamma}{h^3}$$

- A prime issue is the h^3 term
- The resistance of a capillary restrictor is proportional to $(D_{capillary}^4)^{-1}$
- There is the potential for a very high degree of sensitivity to manufacturing tolerances

Theory of Fixed Compensation (contd.)

- For an opposed pad bearing with supply pressure p_s and inlet restrictor resistance R, the total flow is

$$Q = \frac{p_s}{R}$$

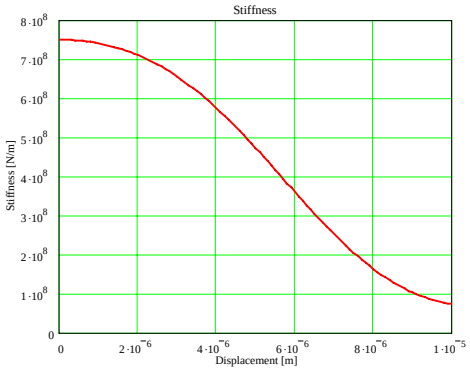
- Stiffness is the change in load for a given change in bearing gap ($A\partial(\Delta P)/\partial\delta$) where $A_{effective}$ is the effective bearing area

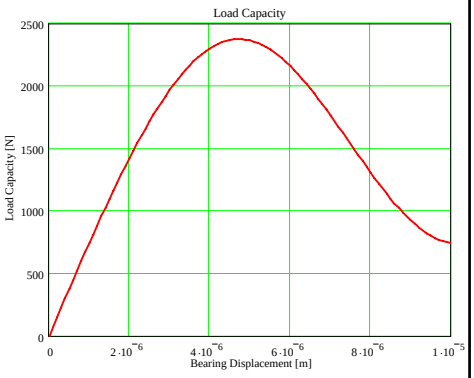
$$k = A_{effective}\frac{\partial \Delta p}{\partial \delta} = 3p_s A_{effective} h^3 \left[\frac{(h-\delta)^2}{\left[(h-\delta)^3 + h^3\right]^2} + \frac{(h+\delta)^2}{\left[(h+\delta)^3 + h^3\right]^2} \right]$$

Example

- Supply pressure p = 2 MPa (= 20 bar)
- Bearing pad width a = 0.05 m
- Bearing pad length b = 0.05 m
- Bearing gap h = 10 μm

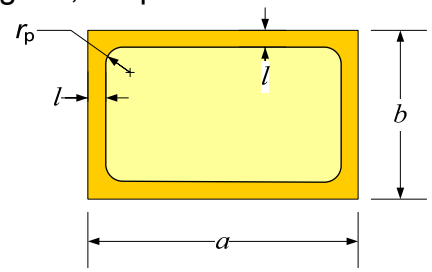
$$A_{effective} = ab = 0.00125m^2$$





Theory of Fixed Compensation (contd.)

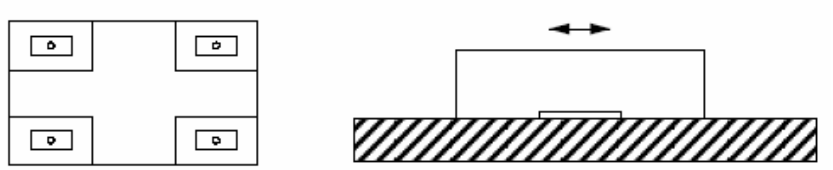
- Bearing effective area consists of the bearing pocket and portions of the land regions
- For a rectangular, flat pad the area is found as:



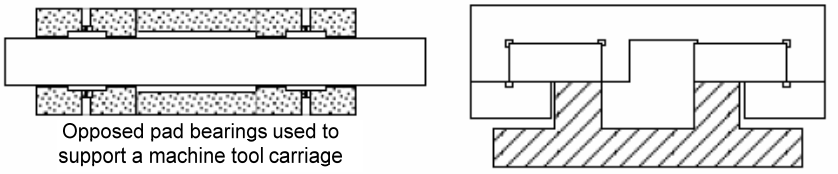
$$A_{effective} = (a-2l)(b-2l) + r_p^2(\pi-4) + l[a+b-4(l+r_p)] + \pi\left[\frac{l(2r_p+l)}{2\ln\left(\frac{r_p+l}{r_p}\right)} - r_p^2\right]$$

Some Examples

Single pad hydrostatic bearings to support a two degree of freedom plate:



Opposed pad bearings used to support a machine toll carriage:

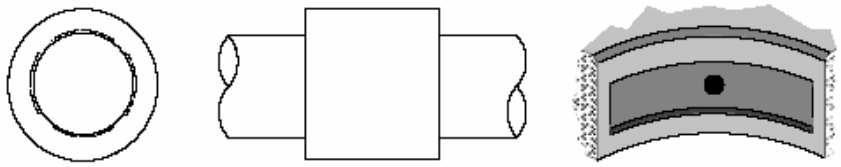


Opposed pad bearings used to support a machine tool carriage

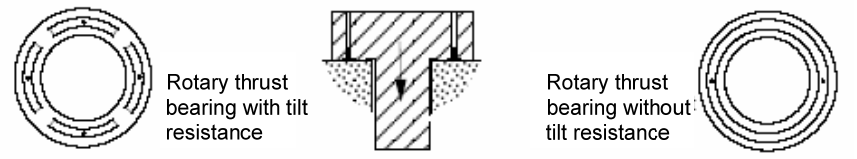
Source: Alexander Slocum, *Precision Machine Design*

Some Examples

Shaft supported by journal bearing:



Rotary thrust bearing:



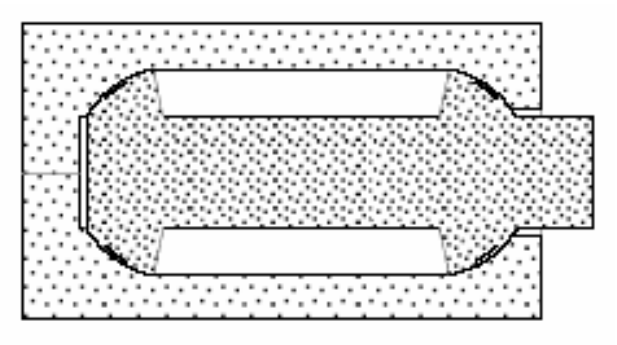
Rotary thrust bearing with tilt resistance

Rotary thrust bearing without tilt resistance

Source: Alexander Slocum, *Precision Machine Design*

Some Examples

Spherical journal and thrust bearing:

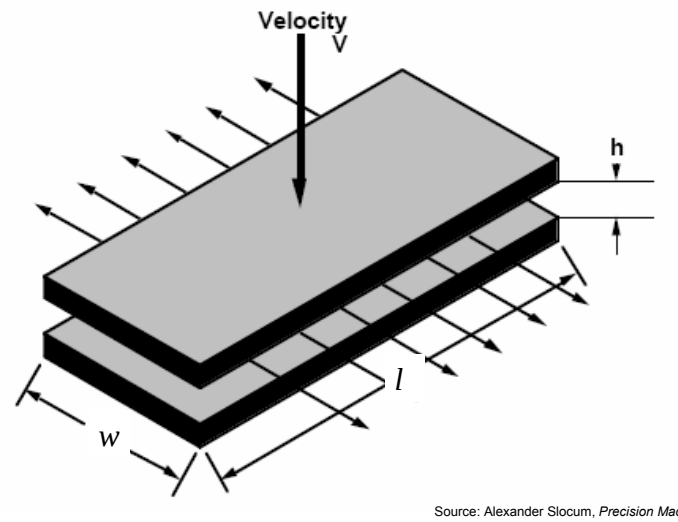


Source: Alexander Slocum, *Precision Machine Design*

Squeeze Film Damping

- The fluid between the lands and the bearing surface is essentially incompressible
- When high frequency (velocity) loads are applied, fluid particles must be squeezed out
- Viscous shear dominates fluid flow out of the land region
- Typical damping coefficients of systems with hydrostatic bearings are ζ = 0.1-1
- Geometric parameters involved in the calculation of the squeeze film damping constant:

Squeeze Film Damping



Source: Alexander Slocum, *Precision Machine Design*

Squeeze Film Damping

- Most of the damping area is rectangular
- The damping factor ξ is calculated as:

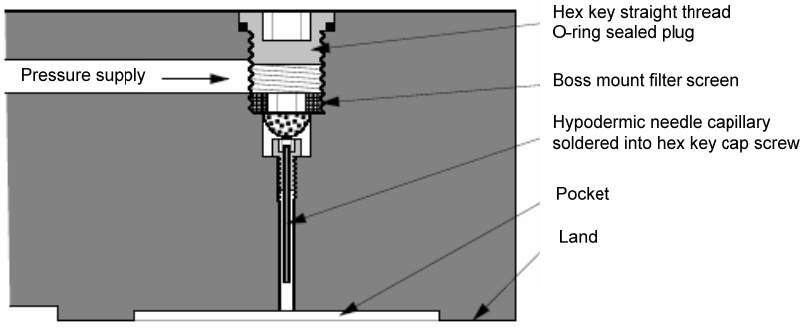
$$\xi = K_s \frac{\mu w^3 l}{h_0^3}$$

K_s is a geometric factor related to the bearing, μ is the viscosity

$$K_s = 0.7925 - \frac{1.1005}{e^{\frac{w}{l}}} + \frac{0.0216}{\frac{w}{l}} + 0.0153\frac{w}{l}$$

Capillary Compensation

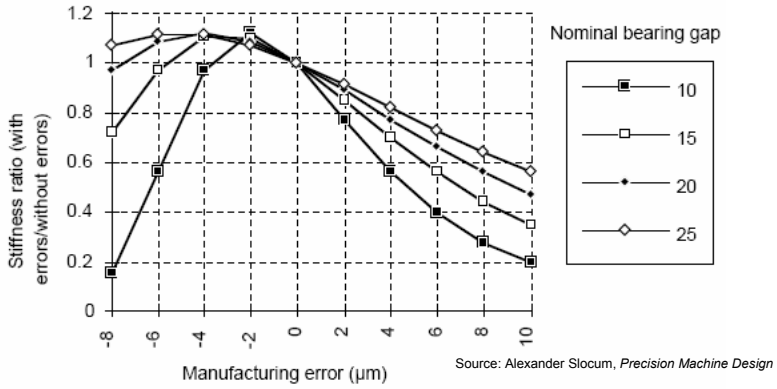
- Opposed pad, capillary restricted bearings are one of the most common hydrostatic bearing designs
- The flow resistance of a capillary is: $R = \frac{8l\mu}{\pi r_c^4}$
- Typical design:



Source: Alexander Slocum, *Precision Machine Design*

Capillary Compensation

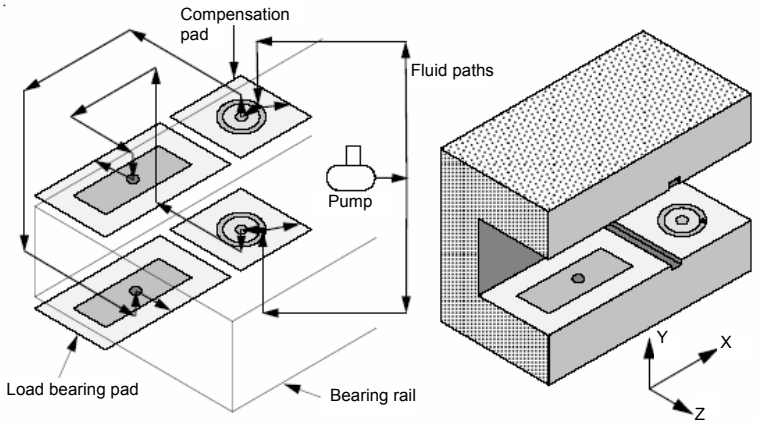
- If the gap should change significantly, however, stiffness can be rapidly lost
- Effect of mfg. errors on capillary compensated bearing:



Source: Alexander Slocum, *Precision Machine Design*

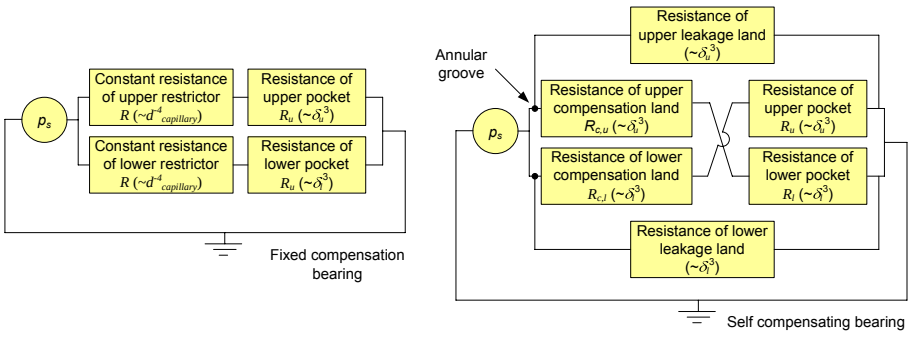
Self Compensation

- The bearing gap itself can be used as a means of regulating the flow of fluid to the opposed bearing



Source: Alexander Slocum, *Precision Machine Design*

Self Compensation



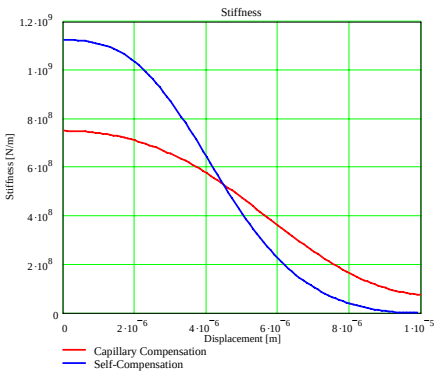
Adapted from: Alexander Slocum, *Precision Machine Design*

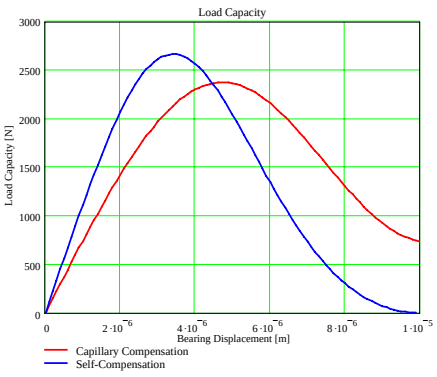
Self Compensation

$$k = 3A_{effective}p_s\left(\begin{array}{l}\dfrac{\dfrac{\gamma}{(h-\delta)^4}-\dfrac{1}{(h+\delta)^4}}{(h+\delta)^3\left(\dfrac{\gamma}{(h-\delta)^3}+\dfrac{1}{(h+\delta)^3}\right)^2}+\dfrac{1}{(h+\delta)^4\left(\dfrac{\gamma}{(h-\delta)^3}+\dfrac{1}{(h+\delta)^3}\right)} \\ -\dfrac{\dfrac{1}{(h-\delta)^4}-\dfrac{\gamma}{(h+\delta)^4}}{(h-\delta)^3\left(\dfrac{1}{(h-\delta)^3}+\dfrac{\gamma}{(h+\delta)^3}\right)^2}+\dfrac{1}{(h-\delta)^4\left(\dfrac{1}{(h-\delta)^3}+\dfrac{\gamma}{(h+\delta)^3}\right)}\end{array}\right)$$

With γ = 3 for self-compensation

Capillary vs. Self-Compensation





p_s = 2E6 Pa a = 50mm
h = 10 μm b = 50 mm

Self Compensation vs. Capillary

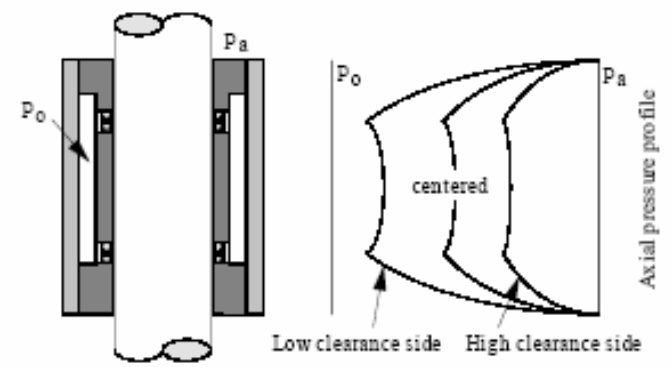
- It makes the system far less sensitive to contamination, especially if water is used
- It provides greater stiffness and load capacity
- It makes the system insensitive to manufacturing tolerances
- The bearings are self-tuning
  - The stiffness automatically optimizes itself for the bearing as soon as it is turned on
- No manual tuning of capillary or orifice size is required

Water vs. Oil

- In addition, it would be preferable to use water (or a water based coolant) instead of oil for the following reasons:
  - Environmentally friendlier
  - No fire hazard
  - Greater heat capacity, which minimizes temperature rise
  - Lower viscosity, which allows for higher speeds
  - Very tolerant of crosstalk between lubrication and coolant systems
- In order to use water, the gap must be small to keep flow rates reasonable
- Self compensated bearings are not significantly affected by large gap variations caused by manufacturing tolerances
  - Thus they are suitable for use with water as a bearing fluid

Aerostatic Bearings

- Aerostatic bearings utilize a thin film of high pressure air (typically 690 kPa) to support a load:



Source: Alexander Slocum, *Precision Machine Design*

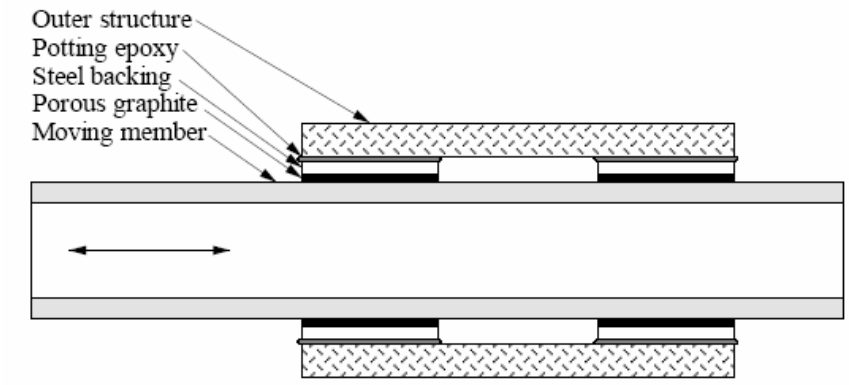
Aerostatic Bearings

- Since air has a very low viscosity, bearings gaps are small, on the order of 1-10 microns
- Aerostatic bearings can be configured in virtually all the ways hydrostatic bearings can
- Because air bearings have essentially zero friction and are very clean running:
  - They have extensive use in metrology equipment and in machines used in clean rooms

Making Aerostatic Bearings

- Modular fluidstatic bearing systems can be easily made from off-the-shelf components:
  - Porous graphite bearing pads (e.g., from Devitt Machinery in Aston PA)
  - Ceramic Hydroguide™ pads from Wilbanks International
  - Hollow alumina beams (e.g., from Wilbanks International in Hillsboro OR)
  - Hollow cast iron square tube (e.g., from Smith Tool Co. in Manasquan, NJ)
  - Epoxy potting compound (with metal fillers to increase modulus) (e.g., DWH epoxy from Devitt Machinery)

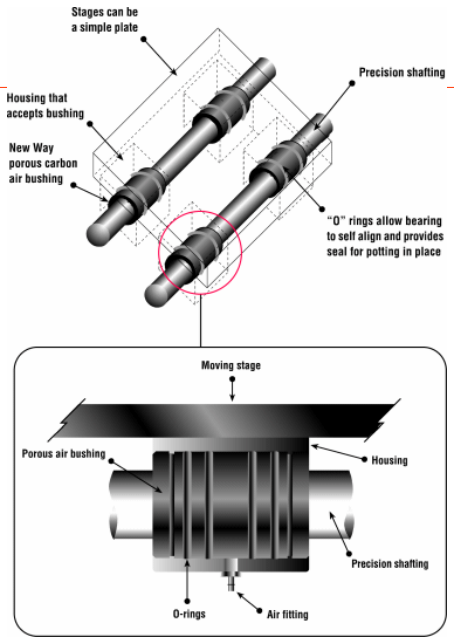
Making Aerostatic Bearings



Source: Alexander Slocum, *Precision Machine Design*

Assembly Procedure

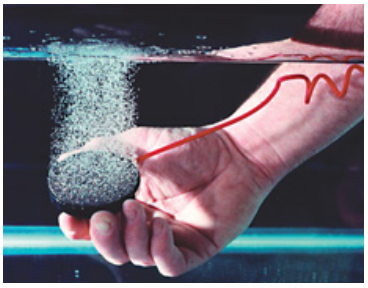
- The outside of the cast iron tube is finish ground
- The modular porous graphite air bearing pads are held to the precision alumina beam by a vacuum
- The beam is positioned with jigs with respect to the outside of the cast iron tube
- The potting epoxy is injected between the pads and the cast iron
- After curing, when the air is off, the alumina beam is locked in place (self-braking)
- When air is applied, the cast iron tube expands a few μm and the alumina beam floats



Source: New Way Airbearings

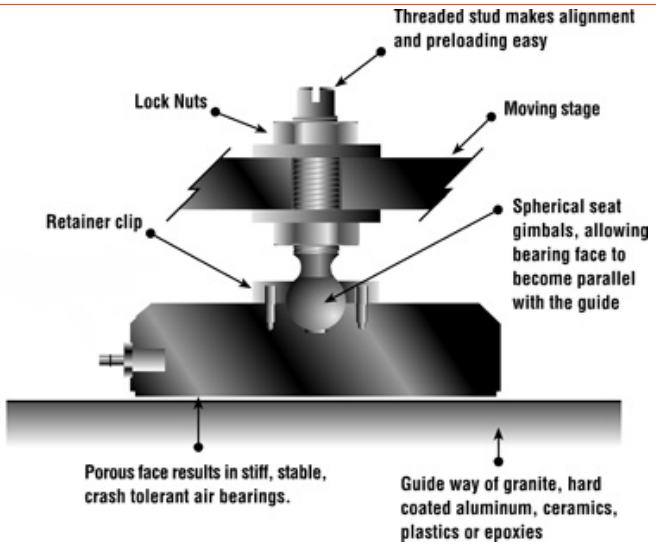
Porous Graphite Bearings

- The fluid film of the bearing is achieved by supplying a flow of air through the bearing itself to the bearing surface
- Unlike traditional ‘orifice' air bearings, porous graphite bearings deliver the air through a porous medium to ensure uniform pressure across the entire bearing area
- The design of the air bearing is such that, although the air constantly dissipates from the bearing site, the continual flow of pressurized air through the bearing is sufficient to support the working loads



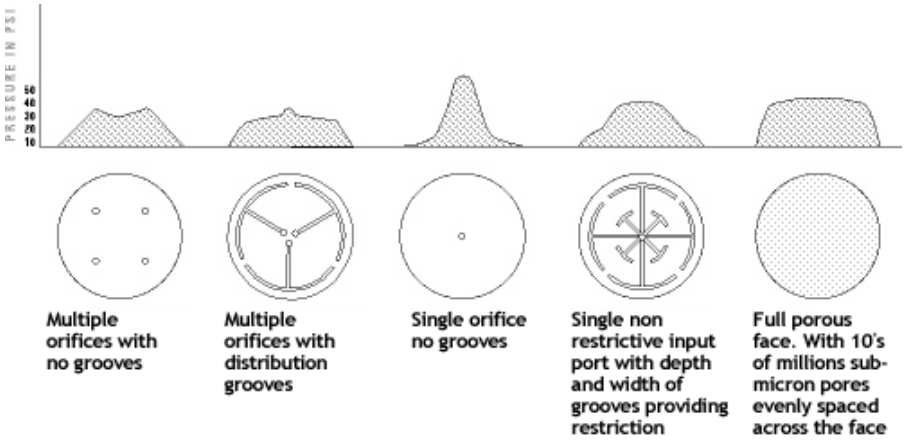
Source: New Way Airbearings

Schematic of a Flat Air Bearing



Source: New Way Airbearings

Pressure Distribution of Air Bearings



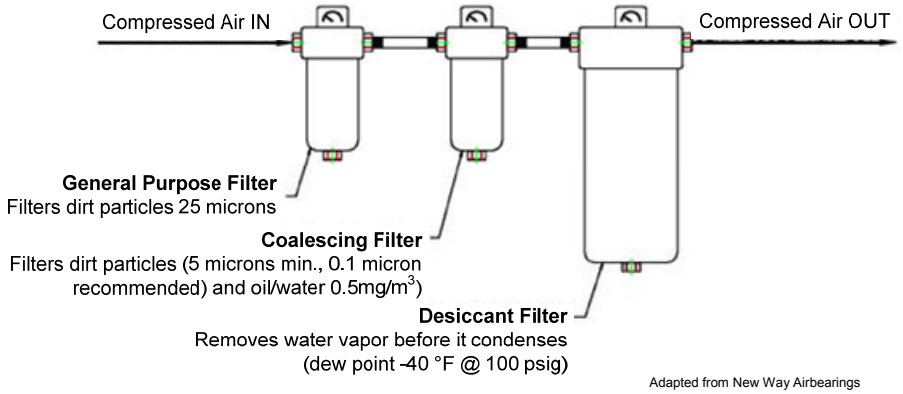
Source: New Way Airbearings

Supply Equipment

- Compressed air used to supply an air bearing must be properly cleaned and dried
- Air bearing performance and useful lifetime greatly depends on the quality of the compressed air
- An efficient system ensures minimum pressure loss, removal of contaminants such us water, oil, dirt, rust, and other foreign materials
- Particles will not affect the performance or life of porous air bearings, but oil and water will
- In order to ensure the specified performance and useful lifetime of the bearings, it is recommended that the following minimum criteria be met:

Source: New Way Airbearings

Supply Equipment



Adapted from New Way Airbearings

Flexures

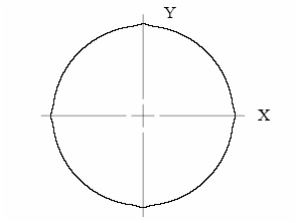
ME EN 7960 – Precision Machine Design
Topic 12

General

- Flexural Bearings are widely used in practice
- Application range from:
  - Flip-top on a shampoo bottle
  - Piezo-beams on a scanning tunneling microscope
- Because of the bending stresses created in the flexure, great care needs to be taken when designing these bearings
- Flexures, if designed properly, are very stiff & robust, light weight, maintenance free, and provide nanometer to sub-nanometer-level guiding precision (simple designs induce cosine runout errors)
- They are frictionless and do neither require lubricants nor air films

Friction in Conventional Linear Bearings

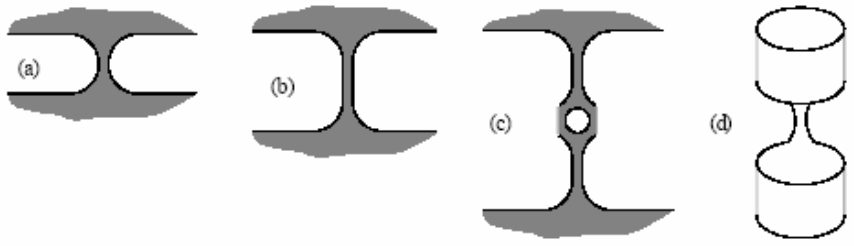
- Static friction approximately equals dynamic friction at low speeds, so stick slip is often minimized
- For heavily loaded tables, static friction is still significantly greater than dynamic friction
  - Errors will appear at velocity crossovers:



Source: Alexander Slocum, *Precision Machine Design*

Bearing	"Dimple" size
Sliding contact	10 – 20 μm
Recirculating rolling element	3 – 5 μm
Crossed rollers	1 – 2 μm
Hydrostatic or aerostatic	0 μm

Monolithic Design



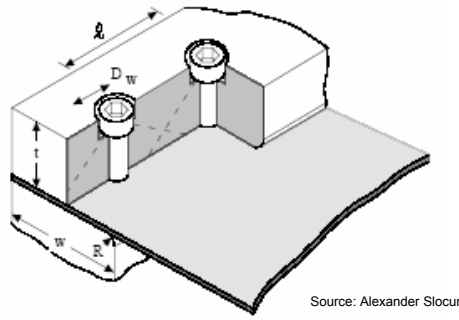
Source: Alexander Slocum, *Precision Machine Design*

- Most accurate
- Sometimes difficult to machine:
  - Abrasive water jet machining is an economical way to cut non-critical areas (links)
  - Wire EDM can be used for precision cuts in critical hinge areas

Monolithic Design

- May require localized heat treating of the material in the flexure
- Applications include flexural couplings, mirror mounts, STM's, and many others
- Size is about 20 times the range of motion

Clamp Design



Source: Alexander Slocum, *Precision Machine Design*

- Nanometer accuracy and better can be obtained if the bolted joint is properly designed
  - Rounded edges
  - Bolt cones-of-action overlap

Clamp Design

- Easy to assemble from annealed parts and hardened spring steel
  - Careful when tightening bolts, as the torque can twist the flexure
    - Lubricated thrust washers can help
- Care is required to make sure residual stresses don't create asymmetries that create parasitic error motions
  - Cut spring metal with EDM or WaterJet
  - Use dual lubricated washers under bolt heads and guides for clamp plates
- Applications include wafer steppers, mirror mounts, and many others
- Size is about 10 times the range of motion

Performance

Speed and acceleration limits

- Limited only by yield strength and design

Range of motion

- Typically used for motions less than a few millimeters
- Monolithic designs: flexure length/motion = 20 or more
- Clamped designs: flexure length/motion = 5-10

Applied loads

- Design goal is to obtain load capacity with minimum spring constant

Repeatability

- Axial: limited only by the drive system
- Lateral: less than nanometers for monolithic designs

Performance

Resolution

- Axial: limited only by the drive system

Preload

- Inherently preloaded

Stiffness

- The greater the motion and the lower the spring rate, the less the stiffness
- Theory of elasticity or finite element analysis yields very accurate predictions of performance
- Flexures often have low transverse stiffness, so they are more susceptible to parasitic forces

Vibration and shock resistance

- Very good

Performance

Damping capability

- Material damping only (2-5%)
- Damping mechanisms (constrained layer dampers) can be used to obtain high damping

Accuracy

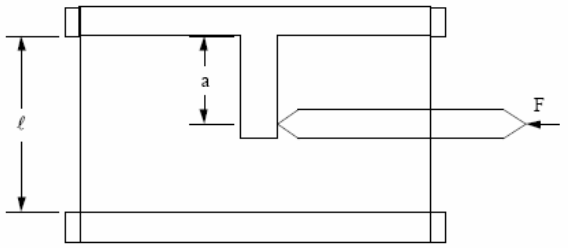
- Axial: limited only by the drive system
- Lateral: can be less than nanometers for monolithic designs
- Depends on how well the bearing was assembled or machined
- Even if there is a small off-axis error motion associated with the primary motion:
  - The error motion is usually very predictable and highly repeatable

Performance

- Flexural bearings cannot attain perfect motion because of:
  - Variation in spring strength
  - Elastic modulus varies with rolling direction in steels
  - Variation in spring geometry
  - Overall inaccuracies of manufacture
  - Bending of the bearing in an unintended manner
  - Bending of structure
  - External applied loads (e.g., gravity and the manner in which the actuation force is applied)

Performance

- The most common errors are the pitch angle and vertical motion
  - They accompany linear motion in a four bar linkage flexure
- For small displacements, the errors are a function of:



Source: Alexander Slocum, *Precision Machine Design*

Performance

- The distance moved x
- The length of the springs l
- The spring thickness t
- The platform length b
- The distance of force application a above the fixed end of the springs

$$\theta_{pitch} = \left(\frac{6(l-2a)t^2}{3b^2l - 2t^2l + 6at^2} \right) \left(\frac{x}{l} \right) \qquad \delta_{vertical} \approx \frac{x^2}{2l}$$

- At $a = l/2$ there is no pitch error.

Performance

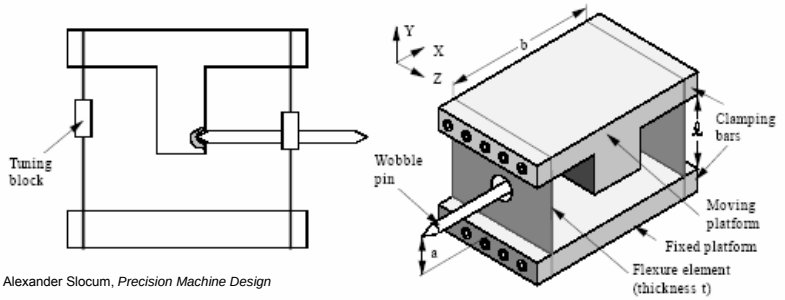
- If the force is applied at a point other than halfway between the platforms:
  - A bending moment is generated which causes a pitch error to occur
- The pitch errors caused by the difference in spring length and difference in platform length are respectively:

$$\theta_{spring} = \frac{\delta_{spring} x^2}{2l^2 b}$$

$$\theta_{platform} = \frac{\delta_{platform} x}{lb}$$

Types of Flexures

- 4 bar linkage



Source: Alexander Slocum, *Precision Machine Design*

- In the y direction there is a small error motion δ_y that is a function of the x motion δ_x and flexure length l:

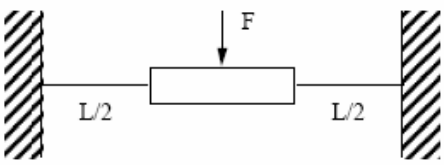
$$\delta_y \approx \frac{\delta_x^2}{2l}$$

Types of Flexures

- Consider other parasitic motions mentioned earlier
- x direction stiffness is equal to two fixed-fixed beams acting together in a side by side mode
- y direction stiffness is equal to that of two columns
- Buckling effect is negligible for small motions and should be prevented by the x direction servo

Minimize Errors

- To minimize parasitic motions:
  - Symmetrical dual four bar linkage eliminates δ_y error
  - Strains along flexure's length resisted by the frame
  - Stiffness is increased: range of motion is decreased



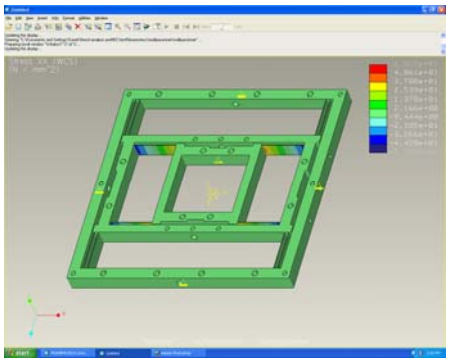
$$\delta = \frac{FL^3}{192EI}$$

Source: Alexander Slocum, *Precision Machine Design*

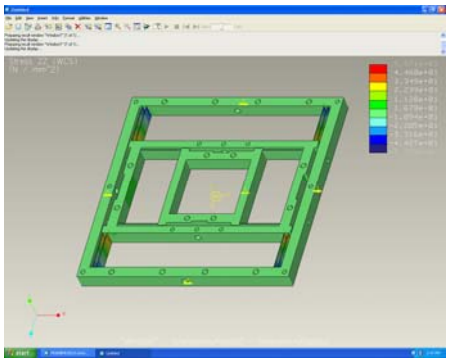
Minimize Errors

- Use a two stage (stacked) four bar linkage:

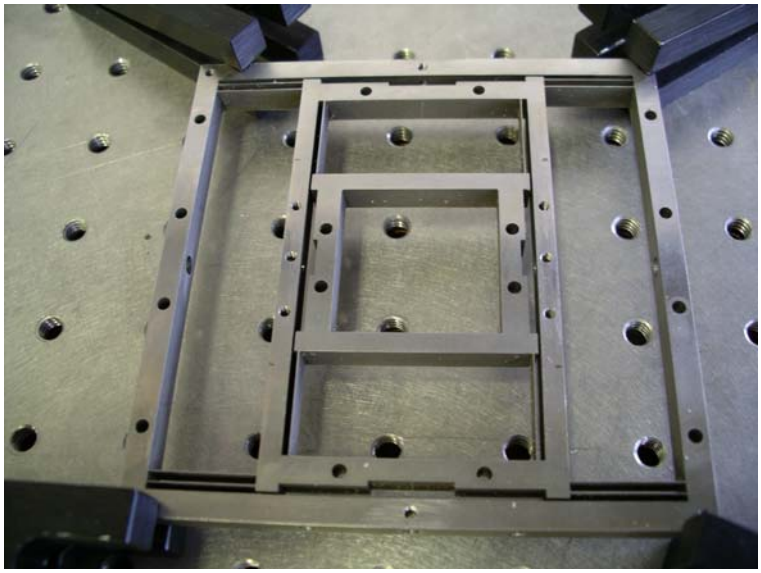
Inner Axis



Outer Axis

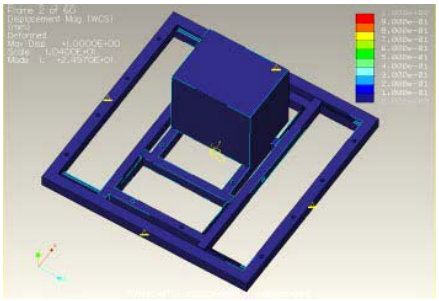


Stacked Axes

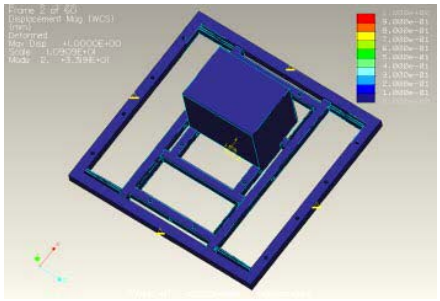


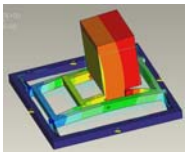
Modal Analysis

Mode 1 at 25.0 Hz

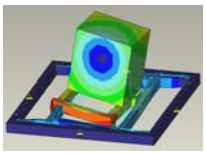


Mode 2 at 33.2 Hz

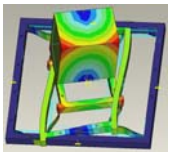




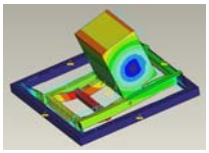
Mode 3: 319.7 Hz



Mode 4: 471.2 Hz

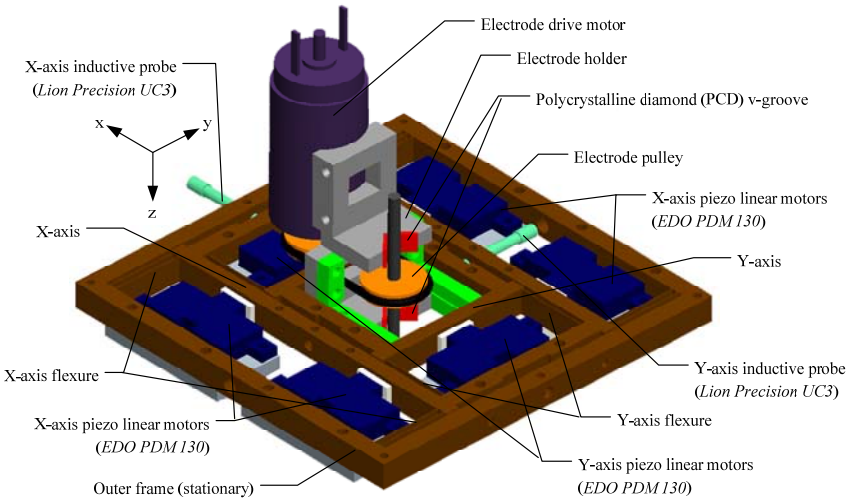


Mode 5: 516.5 Hz



Mode 6: 642.9 Hz

Design of Flexural X-Y Micro-EDM Head



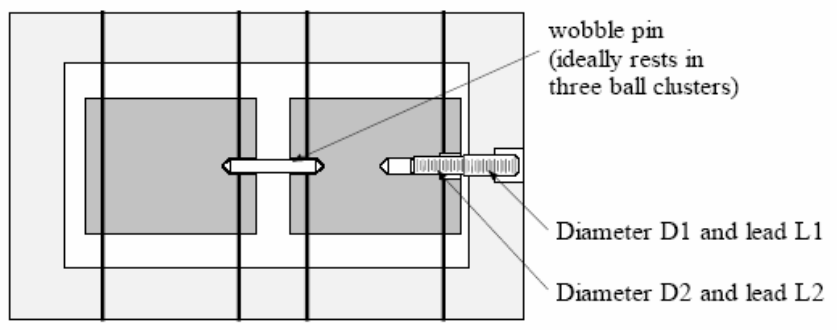
Flexural 2-axis EDM head with piezo linear motors and non-contact inductive probes

Minimize Errors

- The parasitic motion of one 4-bar cancels the parasitic motion of the second 4-bar
- Lateral and yaw stiffness will not be high, however, due to buckling effect
- Overcome by use of monolithic hourglass-type members

Minimize Errors

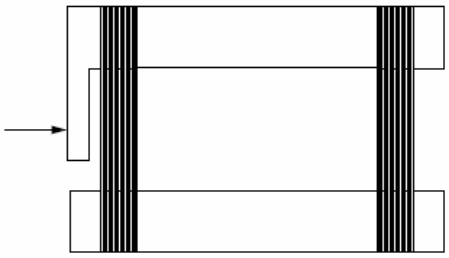
- A differential actuation system can be used to increase resolution
  - A wobble pin will minimize parasitic force actuation errors



Source: Alexander Slocum, *Precision Machine Design*

Minimize Errors

- Many thin blades can be used to increase lateral stiffness and load capacity while keeping axial stiffness and stress low



Source: Alexander Slocum, *Precision Machine Design*

$$F_{load\ capacity} \approx Ntw \qquad k_{axial\ multiblade} = \frac{L^3}{ENt^3w}$$

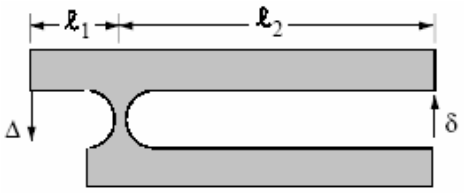
$$k_{vertical} = \frac{NtwE}{L} \qquad k_{axial\ monolithic} = \frac{L^3}{E(Nt)^3w}$$

Minimize Errors

- Be careful of alignment and slip problems between the blades
- EDM process can be used to prevent these problems
- Rubber placed between the blades can increase damping
- Careful when tightening bolts, as the torque can twist the flexure

Flexural Transmission Systems

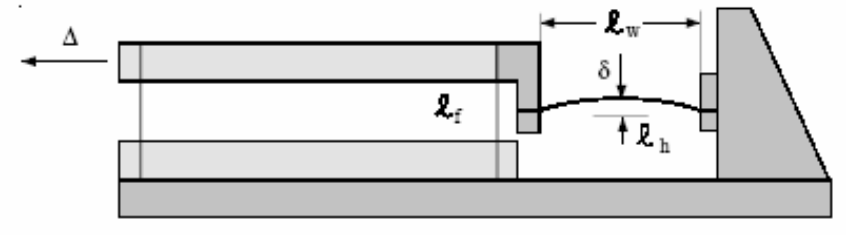
- Lever and fulcrum transmission:



Source: Alexander Slocum, *Precision Machine Design*

Flexural Transmission Systems

- A bowed flexure used as a high reduction transmission:



- A downward motion δ causes a lateral motion Δ:

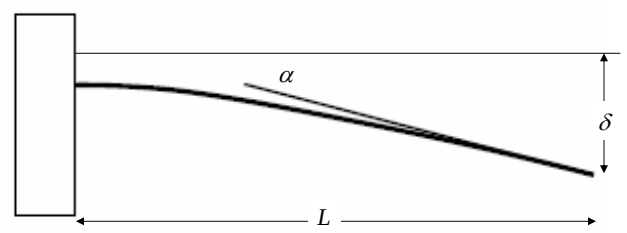
$$\Delta \approx \frac{4\delta l_h}{l_w}$$

Flexural Transmission Systems

- It is possible to chain together a series of these bowed flexures at right angles to each other
  - This can yield a very high transmission ratio
  - Assume that l_h = 2 mm and l_w = 20 mm, then the transmission ratio is 5
  - In series, the ratios become 25, 125, 625... for 2, 3, 4... units respectively
- A variation to this bowed beam approach is to use two beams laid on top of each other and tied together at one end
- When the beams bend there will be differential motion along the interface between them

Angular Flexures

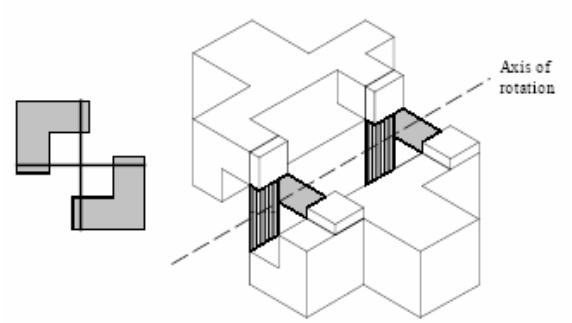
- The slope at the end of the beam causes an Abbe error in the beam that holds the optic which cancels the beam deflection $\delta = \alpha(2/3L)$:



$$\delta = \frac{FL^3}{3EI} \qquad \alpha = \frac{FL^2}{2EI}$$

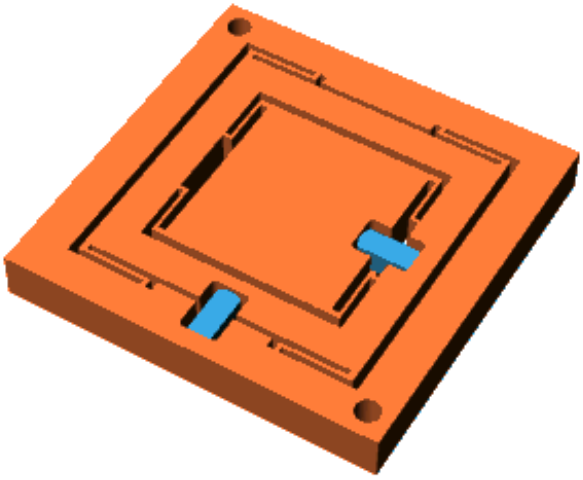
Angular Flexures

- Cross-strip flexure for angular motion:



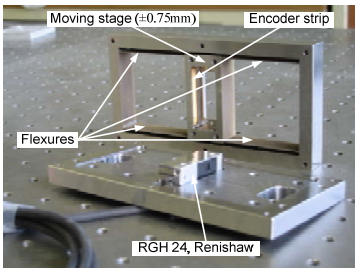
Source: Alexander Slocum, *Precision Machine Design*

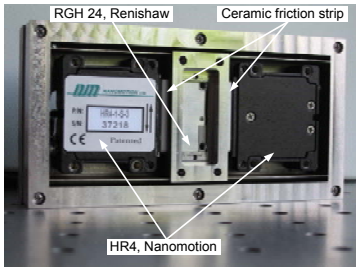
Application Examples

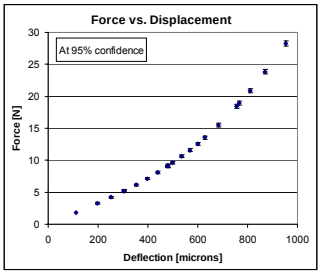


Source: Physikinstrumente at www.physikinstrumente.de

Piezo-Driven Linear Axis







Design of Machine Structures

ME EN 7960 – Precision Machine Design
Topic 14

Topics

- Overall design approach for the structure
- Stiffness requirements
- Damping requirements
- Structural configurations for machine tools
- Other structural system considerations

Design Strategies

- Strategies for Accuracy:
  - Accuracy obtained from component accuracy
    - Most machine tools are built this way
  - Accuracy obtained by error mapping
    - Most coordinate measuring machines are built this way
  - Accuracy obtained from a metrology frame
    - Special machines are built this way (usually one-of-a-kind cost-is-no-object machines)
- Kinematic design:
  - Deterministic
  - Less reliance on manufacturing
  - Stiffness and load limited, unless pot in epoxy

Design Strategies (contd.)

- Elastically averaged design:
  - Non-deterministic
  - More reliance on manufacturing
  - Stiffness and load not limited
- Passive temperature control:
  - Minimize and isolate heat sources
  - Minimize coefficient of thermal expansion
  - Maximize thermal diffusivity
  - Insulate critical components
  - Use indirect lighting
  - Use PVC curtains to shield the machine from infrared sources

Design Strategies (contd.)

- Active temperature control:
  - Air showers
  - Circulating temperature controlled fluid
  - Thermoelectric coolers to cool hot spots
  - Use proportional control
- Structural configurations:
  - Where are the center of mass, friction and stiffness located?
  - What does the structural loop look like?
  - Open frames (G type)
  - Closed frames (Portal type)
  - Spherical (NIST's M3)
  - Tetrahedral (Lindsey's Tetraform)
  - Hexapods (Stewart platforms)
  - Compensating curvatures
  - Counterweights

Design Strategies (contd.)

- Damping:
  - Passive:
    - Material and joint-μslip damping
    - Constrained layers, tuned mass dampers
  - Active:
    - Servo-controlled dampers (counter masses)
    - Active constrained layer dampers

Summary of Strategies for Accuracy

- Accuracy obtained from component accuracy:
  - Inexpensive once the process is perfected
  - Accuracy is strongly coupled to thermal and mechanical loads on the machine
- Accuracy obtained by error mapping:
  - Inexpensive once the process is perfected
  - Accuracy is moderately coupled to thermal and mechanical loads on the machine
- Accuracy obtained from a metrology frame:
  - Expensive, but sometimes the only choice
  - Accuracy is uncoupled to thermal and mechanical loads on the machine

Stiffness Requirements

- Engineers commonly ask "how stiff should it be?"
- A minimum specified static stiffness is a useful but not sufficient specification
- Static stiffness and damping must be specified
- Static stiffness requirements can be predicted
- Damping can be specified and designed into a machine

Minimum Static Stiffness

- For heavily loaded machine tools, the required stiffness may be a function of cutting force
- For lightly loaded machines and quasi-statically positioning, use the following:
  - First make an estimate of the system's time constant:

$$\tau_{mech} = 2\pi\sqrt{\frac{m}{k}}$$

  - The control system loop time τ_{loop} must be at least twice as fast to avoid aliasing
- Faster servo times create an averaging effect by the factor $(\tau_{mechanical}/2\tau_{loop})^{½}$

Minimum Static Stiffness (contd.)

- For a controller with N bits of digital to analog resolution, the incremental force input is:

$$\Delta F = \frac{F_{\max}}{2^N\sqrt{\frac{\tau_{mech}}{2\tau_{servo}}}}$$

- The minimum axial stiffness is thus:

$$k \geq \left(\frac{F_{\max}\tau_{servo}^{1/2}}{2^N\pi^{1/2}m^{1/4}\delta_K}\right)^{4/3}$$

Minimum Static Stiffness (contd.)

- While the controller is calculating the next value to send to the DAC, the power signal equals the last value in the DAC
- The motor is receiving an old signal and is therefore running open loop
- Assume that there is no damping in the system
- The error δ_M due to the mass being accelerated by the force resolution of the system for a time increment τ_{servo} is

$$\delta_M = \frac{1}{2}\frac{\Delta F}{M}\tau_{servo}^2$$

Minimum Static Stiffness (contd.)

- The maximum allowable servo-loop time is thus

$$\tau_{servo} = \sqrt{\frac{2\delta_M M}{\Delta F}}$$

- The minimum axial stiffness is thus:

$$K \geq \frac{F_{\max}\delta_M^{1/4}}{2^{(N-0.25)}\pi^{1/2}\delta_K^{5/4}}$$

- It must also be greater than the stiffness to resist cutting loads or static loads not compensated for by the servos:

$$K \geq \frac{F_{\max}}{\delta}$$

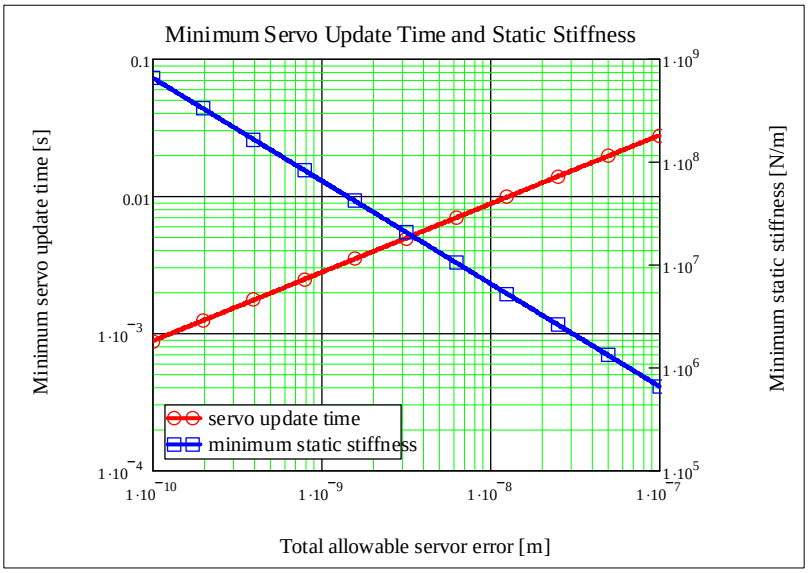
Minimum Static Stiffness (contd.)

- The maximum servo-loop time is thus:

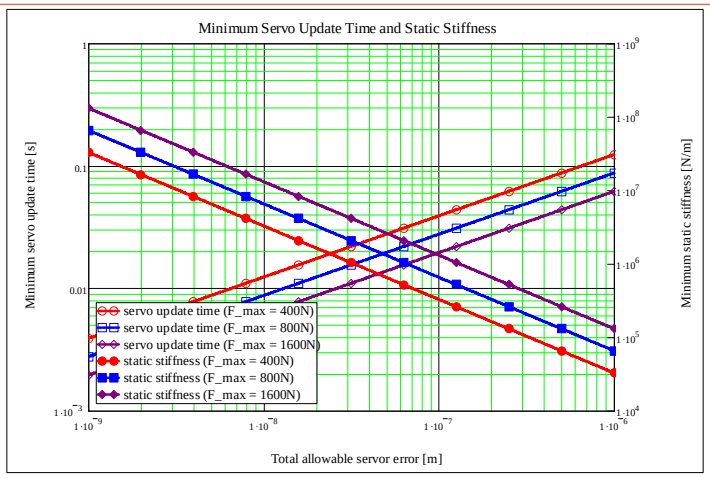
$$\tau_{servo} \le \sqrt{\frac{\pi^{1/2} 2^{(N+0.75)} \delta_M^{3/4} m \delta_K^{1/4}}{F_{\max}}}$$

- Typically, one would set $\delta_K = \delta_M = \frac{1}{2}\delta_{servo}$
- Usually, $\tau_{servo\ actual} = \tau_{servo}/L$, where L is the number of past values used in a recursive digital control algorithm
- Example: Required static stiffness for a machine with 800 N max. axial force, 250 kg system mass, and 14 bit DAC:

Example

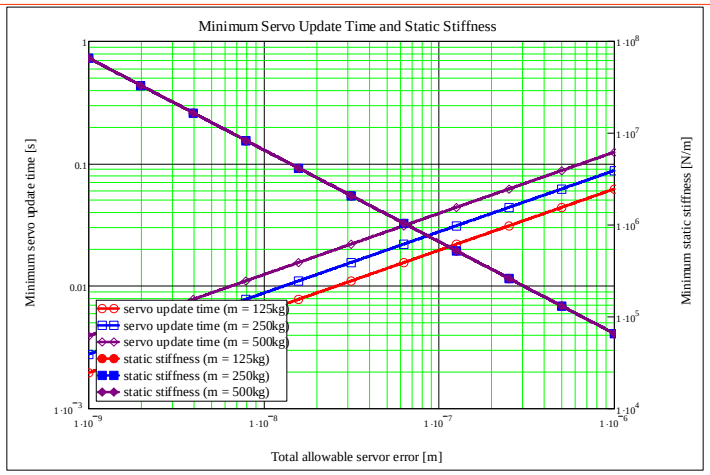


Servo System Force Output



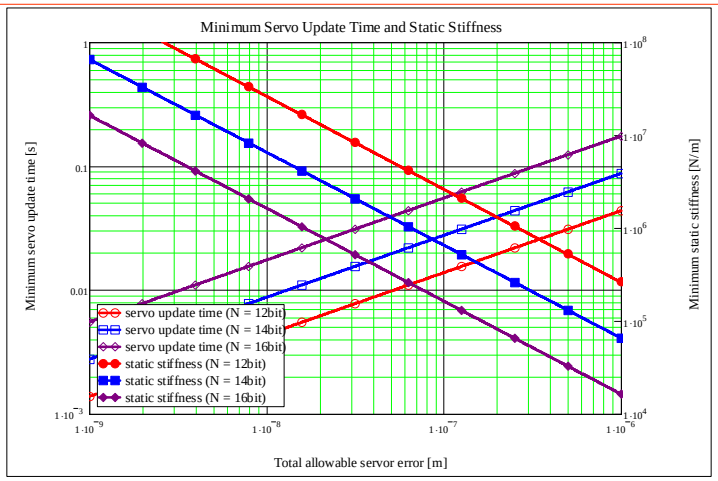
- Lower force drive system for a given servo error
  - Increases the servo update time
  - Lowers the static stiffness requirement

System Mass



- Lower mass
  - Decreases the servo update time
  - Does not affect static stiffness requirement

Servo DAC Resolution

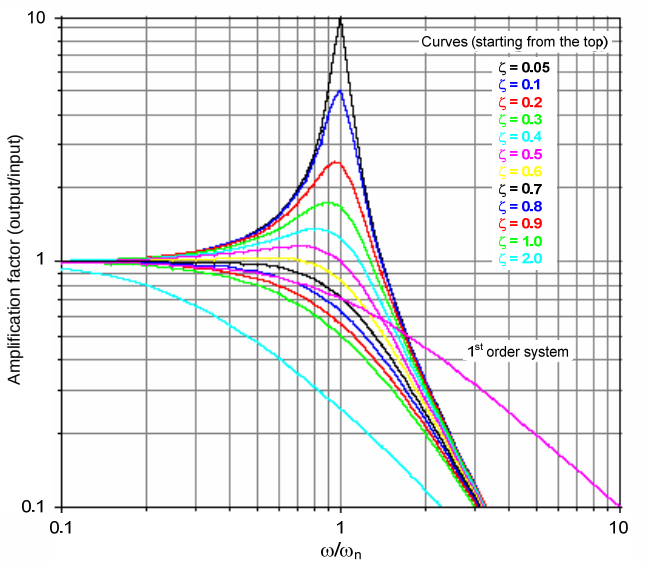


- Lower DAC resolution
  - Decreases the servo update time
  - Increases static stiffness requirement

Dynamic Stiffness

- Dynamic stiffness is a necessary and sufficient specification
- Dynamic stiffness:
  - Stiffness of the system measured using an excitation force with a frequency equal to the damped natural frequency of the structure
- Dynamic stiffness can also be said to be equal to the static stiffness divided by the amplification (Q) at resonance

Dynamic Stiffness (contd.)



It takes a lot of damping to reduce the amplification factor to a low level.

Source: Alexander Slocum, *Precision Machine Design*

Dynamic Stiffness (contd.)

- Material and joint damping factors are difficult to predict and are too low anyway.
- For high speed or high accuracy machines:
  - Damping mechanisms must be designed into the structure in order to meet realistic damping levels.
- The damped natural frequency and the frequency at which maximum amplification occurs are

$$\omega_d = \omega\sqrt{1-\zeta^2}$$

$$\omega_{d\ peak} = \omega\sqrt{1-2\zeta^2}$$

Dynamic Stiffness (contd.)

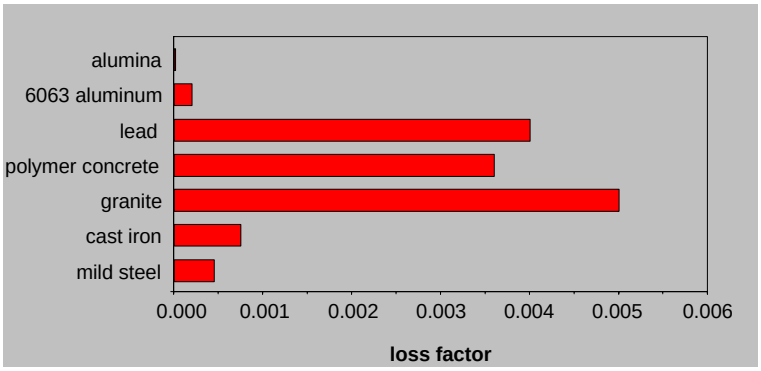
- The amplification at the damped natural frequency and the peak frequency can thus be shown to be

$$Q = \frac{Output}{Input} = \frac{1}{\sqrt{4\zeta^2 - 3\zeta^4}}$$

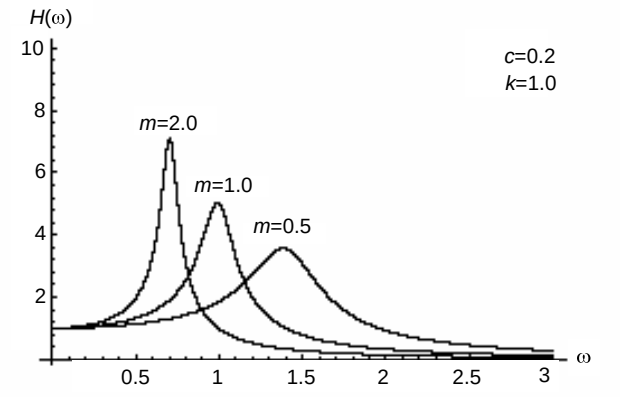
$$Q\frac{Output_{peak}}{Input} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{k_{static}}{k_{dynamic}} \approx \frac{1}{2\zeta}$$

- For unity gain or less, ζ must be greater than 0.707
- Cast iron can have a damping factor of 0.0015
- Epoxy granite can have a damping factor of 0.01-0.05
- All the components bolted to the structure (e.g., slides on bearings) help to damp the system
- To achieve more damping, a tuned mass damper or a shear damper should be used

Material Damping



Effects of Changing System Mass



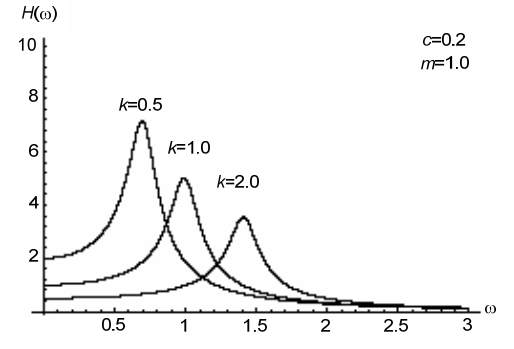
Source: Alexander Slocum, *Precision Machine Design*

- Adding mass:
  - Adding sand or lead shot increases mass and damping via the particles rubbing on each other
  - Higher mass slows the servo response, but helps attenuates high frequency noise

Decreasing mass

- Faster respond to command signals
- Increases a higher natural frequency
  - Higher speed controller signals must be used
- Improved damping, a result of the increased loss factor (the loss factor $\zeta = c/(2m)$
- However, low mass systems show less noise rejection at higher frequencies
  - This suggests that the machine will be less able to attenuate noise and vibration

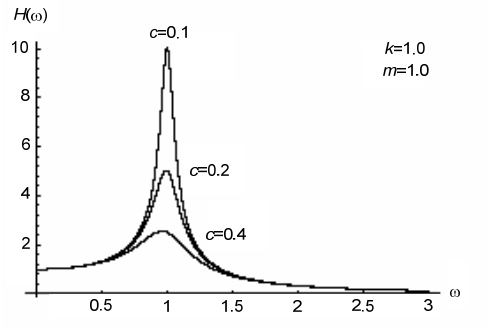
Effects of Adding Stiffness to the Machine System



Source: Alexander Slocum, *Precision Machine Design*

- Higher stiffness gives a flatter response at low frequencies and give smaller displacements for a given force input
- The compromise of decreased noise attenuation is not as dramatic as is the case with lowering the system mass
  - This is shown by the similar shapes in the three response curves at high frequencies
- This suggests that raising the stiffness of a system is always a desirable course of action
  - However, acoustical noise may be worsened by adding stiffness (frequency of vibration is moved to the audible region of the human ear)

Effects of Adding Damping to the Machine System



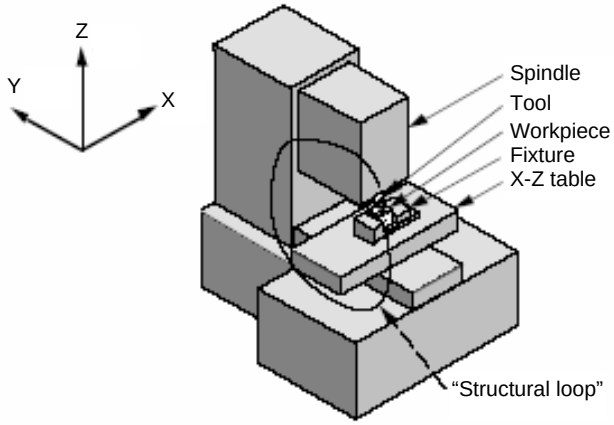
Source: Alexander Slocum, *Precision Machine Design*

- Increasing the system damping can make a dramatic improvement in the system response
- The trend is for decreasing amplification of the output at resonance with increasing damping
- The plot shows the dramatic improvement available by doubling the system damping:
  - Although a damping coefficient of 0.4 may be difficult to obtain in practice

Summary

- For a servo controlled machine:
  - The stiffness of the machine structure should be maximized to improve positioning accuracy
  - The mass should be minimized to reduce controller effort and improve the frequency response and loss factor (ζ)
  - Damping, however, must be present to attenuate vibration in the machine system

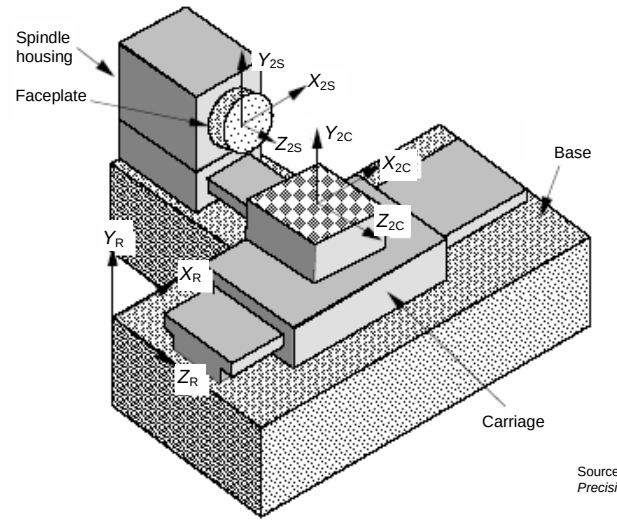
Open Frame Structures



- Easy access to work zone
- Structural loop prone to Abbe errors (like calipers!)

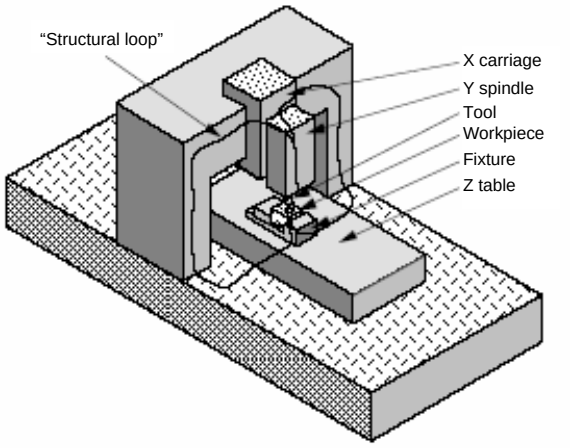
Source: Alexander Slocum, *Precision Machine Design*

Open Frame Structures (contd.)



Source: Alexander Slocum, *Precision Machine Design*

Closed Frame Structures



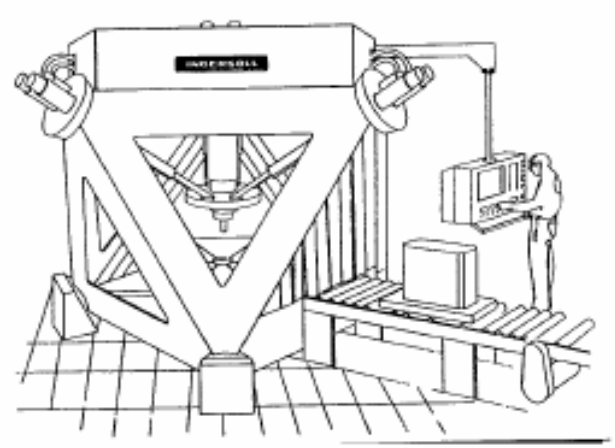
- Moderately easy access to work zone
- Moderately strong structural loop (like a micrometer!)
- Primary/follower actuator often required for the bridge
- Easier to obtain common centers of mass, stiffness, friction

Source: Alexander Slocum, *Precision Machine Design*

Closed Frame Structures

Source: Precision Design Lab

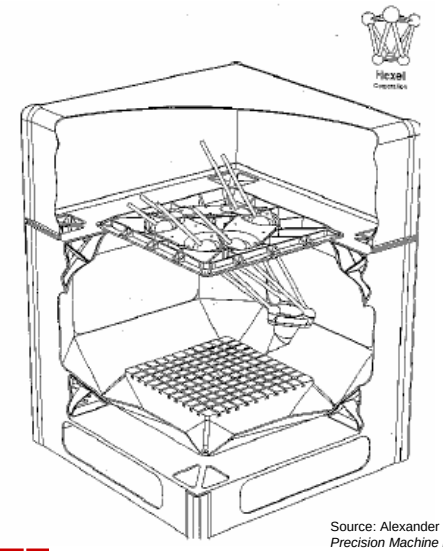
Tetrahedral Structures



- Composed of six legs joined at spherical nodes
- Work zone in center of tetrahedron
- Bearing ways bolted to legs
- High thermal stability
- High stiffness
- Viscous shear damping mechanisms built into the legs
- Damping obtained at the leg joints by means of sliding bearing material applied to the self centering spherical joint
- Inherently stable shape

Source: Alexander Slocum, *Precision Machine Design*

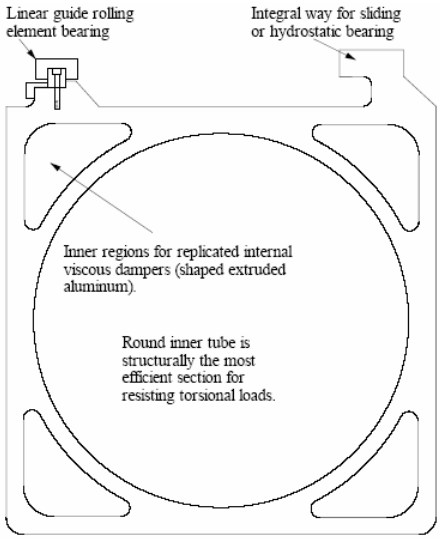
Tetrahedral Structures



- Like the tetrahedron, the octahedron is a stable truss-type geometry (comprised of triangles)
- As the work volume increases, the structure grows less fast than a tetrahedron
- The hexapod (Stewart platform concept originally developed for flight simulators) gives six limited degrees of freedom
- The tool angle is limited to about 20 degrees from the vertical
- Advanced controller architecture and algorithms make programming possible

Source: Alexander Slocum, *Precision Machine Design*

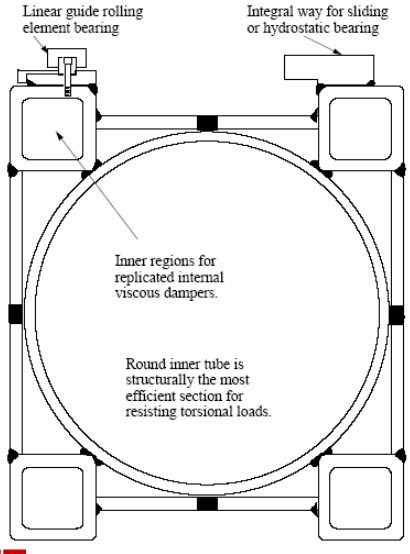
Cast Iron Structures



- Widely used
- Stable with thermal anneal, aging, or vibration stress relieve
- Good damping and heat transfer
- Modest cost for modest sizes
- Integral ways can be cast in place
- Design rules are well established (see text)
- Economical in medium to large quantities

Source: Alexander Slocum, *Precision Machine Design*

Welded Structures



- Often used for larger structures or small-lot sizes
- Stable with thermal anneal
- Low damping, improved with shear dampers
- Modest cost
- Integral ways can be welded in place
- Structures can be made from tubes and plates

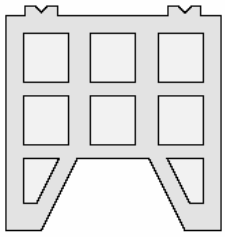
Source: Alexander Slocum, *Precision Machine Design*

Epoxy Granite Structures

- Can be cast with intricate passages and inserts
- $E_{\text{epoxy granite}}/E_{\text{cast iron}}$ = 5/20
- Exterior surface can be smooth and is ready to paint
- Cast iron or steel weldments can be cast in place, but beware of differential thermal expansion effects
- Epoxy granite's lower modulus, and use of foam cores means that local plate modes require special care when designing inserts to which other structures are bolted
- Sliding contact bearing surfaces can be replicated onto the epoxy granite

Epoxy Granite Structures

- Foam cores reduce weight:

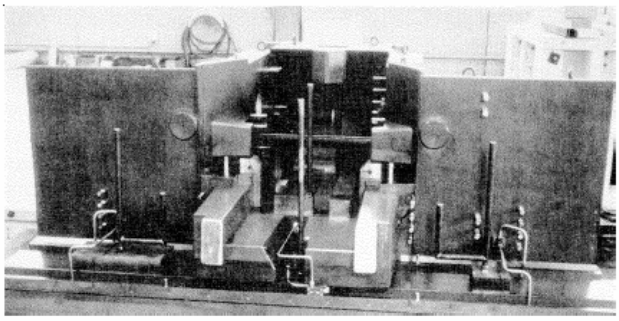


Source: Alexander Slocum, *Precision Machine Design*

- For some large one-of-a-kind machines
- A mold is made from thin welded steel plate that remains an integral part of the machine after the material is cast
- Remember to use symmetry to avoid thermal warping
- Consider the effects of differential thermal expansion when designing the steel shell
- The steel shell should be fully annealed after it is welded together

Epoxy Granite Structures

- Instead of ribs, polymer concrete structures usually use internal foam cores to maximize the stiffness to weight ratio
- Polymer concrete castings can accommodate cast in place components (Courtesy of Fritz Studer AG.)



Source: Alexander Slocum, *Precision Machine Design*

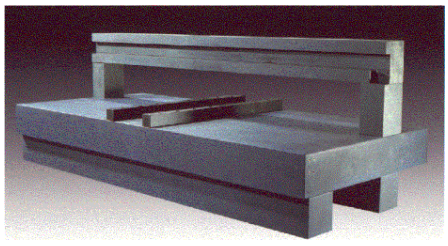
Epoxy Granite Structures

- With appropriate section design:
  - Polymer concrete structures can have the stiffness of cast iron structures
  - They can have much greater damping
  - Highly loaded machine substructures (e.g. carriages) are still best made from cast iron
- Polymer concrete does not diffuse heat as well as cast iron
  - Attention must be paid to the isolation of heat sources to prevent the formation of hot spots
- When bolting or grouting non-epoxy granite components to an epoxy granite bed, consider the bi-material effect

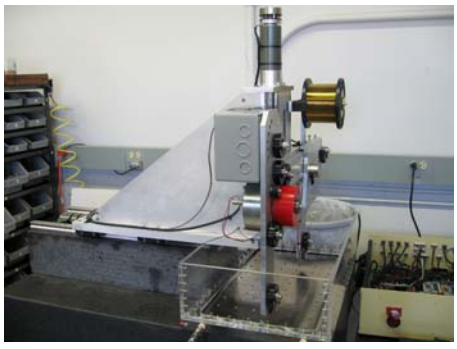
Granite

- Dimensionally very stable
- Must be sealed to avoid absorption of water
- Can be obtained from a large number of vendors providing excellent flatness and orthogonality
- Cannot be tapped, therefore bolt holes consist of steel plugs that have been potted in place that are drilled and tapped after the epoxy has cured
- Can chip
- Provides excellent damping

Granite (contd.)

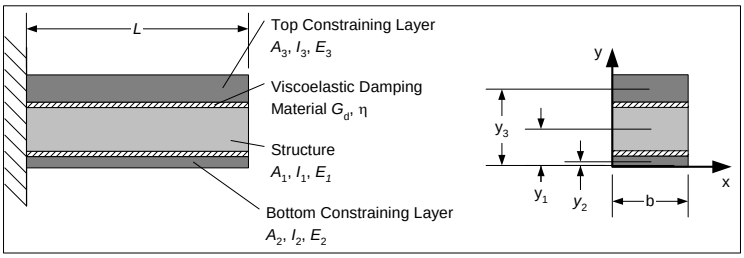


Source: Standridge Granite



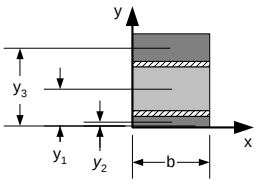
Source: Precision Design Lab

Constrained Layer Damping
How does it work?



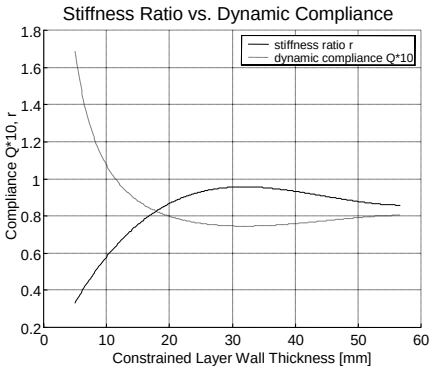
- Visco-elastic layer damps motion between structure and constraining layer (from bending or torsion) by dissipating kinetic energy into heat

Design parameters to tune damper

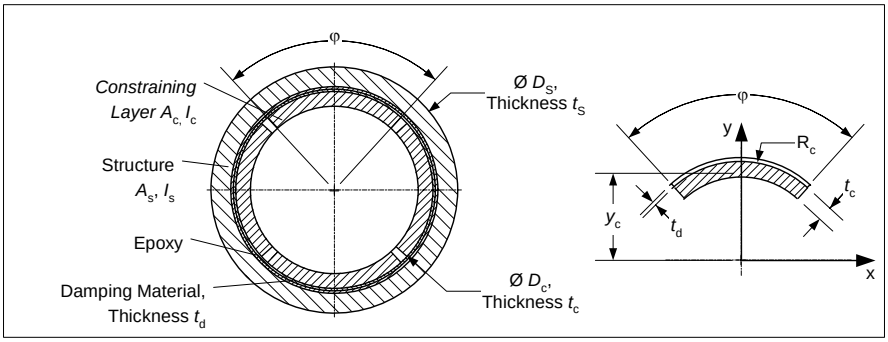


$$EI_\infty = EI_0 + \sum_i E_i A_i (y_i^2 - y_\infty^2)$$

$$r = \frac{EI_\infty}{EI_0} - 1$$



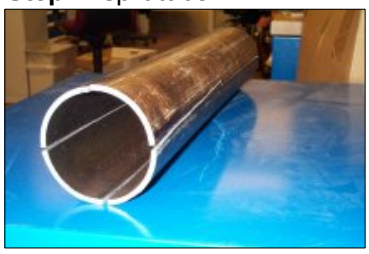
How is it implemented?



- For round structures, inner tube serves as constraining layer (ShearDamper™)
- Constraining layer is wrapped with damping material
- Coated inner tube is inserted and gap filled with epoxy

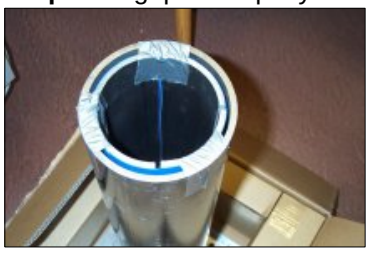
ShearDamper™ - step by step

Step 1: split tube



Step 2: wrap damping sheet around

Step 3: fill gap with epoxy



Step 4: done

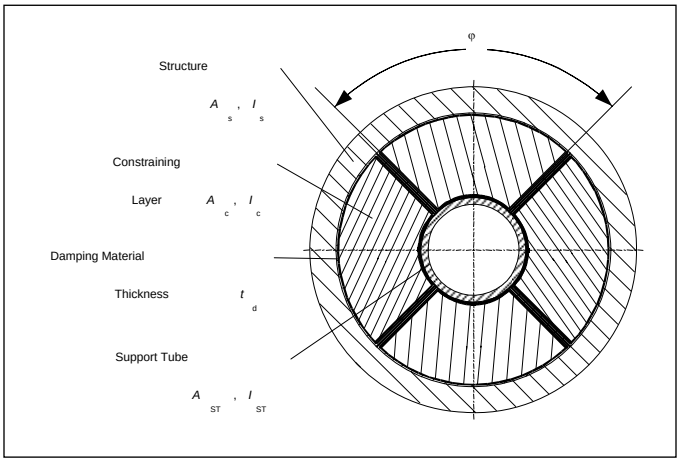
Any tradeoffs?

- Labor intensive – inner tube needs to be split
- Inner tube and epoxy are expensive
- Added weight lowers modal frequencies
- Challenging if bottom is not accessible for sealing
- Constraining layer performance depends on available wall thickness
- Only works for round or rectangular structures where a matching split tube is available

How can we make it better?

- Replace steel tube AND epoxy with cheaper material that has better internal damping
- Make the need for splitting the constraining layer obsolete
- Make design more flexible in terms of shape and required stiffness
- Remove design constraints

But how?



- Four “sausage-like” damping sleeves are inserted between the outer structural and the inner support tube
- Dampers are filled with expanding concrete

E. Bamberg, A.H. Slocum, “Concrete-based constrained layer damping”, Precision Engineering, 26(4), October 2002, pp. 430-441.

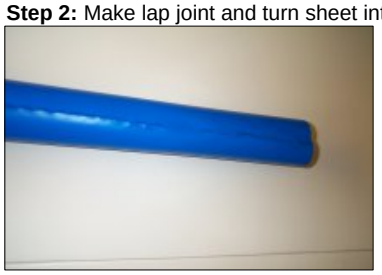
Concrete cast – it’s simple!

- Flexible constraining layer thickness – wide range of standard tubes can be used as support tube, wall thickness is no longer a design constraint
- Concrete provides additional damping
- Cheap
- Fast

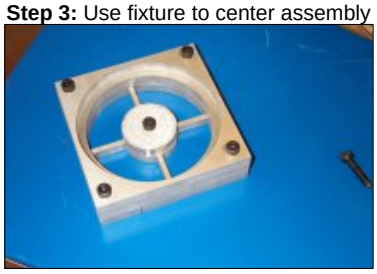
How is it done – part 1?

Step 1: Cut damping sheet

Step 2: Make lap joint and turn sheet into tube



Step 3: Use fixture to center assembly



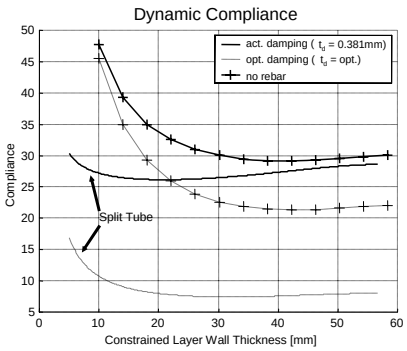
Step 4: Seal bottom with cable ties

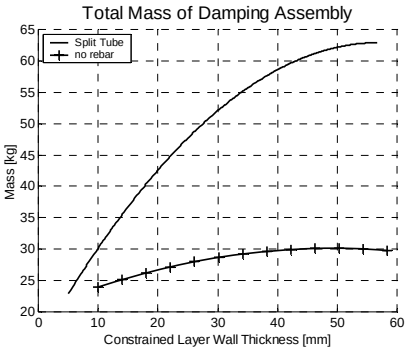
How is it done – part 2?

Step 5: Pour expanding concrete

Step 6: After concrete has cured – cut off ends: done

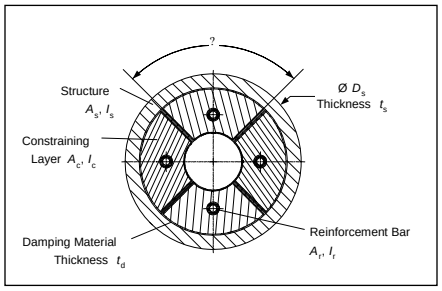
Dynamic compliance - lower is better…

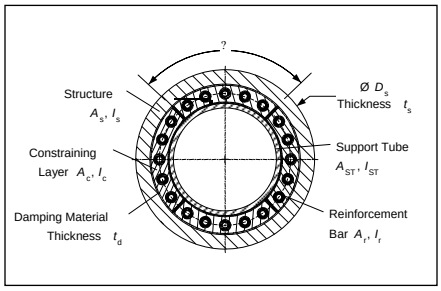




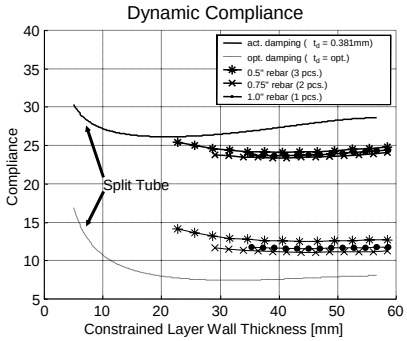
Steel constraining layer is better because it has a higher Young's modulus than concrete – an equally stiff layer is thinner and hence further away from system neutral axis → increased stiffness ratio *r*.

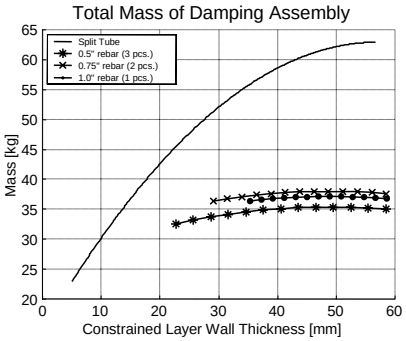
Reinforced works even better…



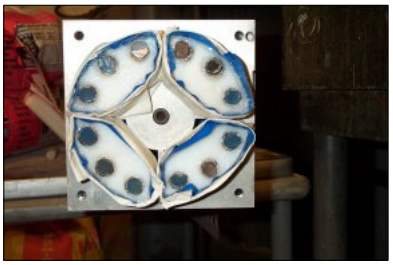


Concrete cast rocks! (Virtually…)





Reinforced concrete cast

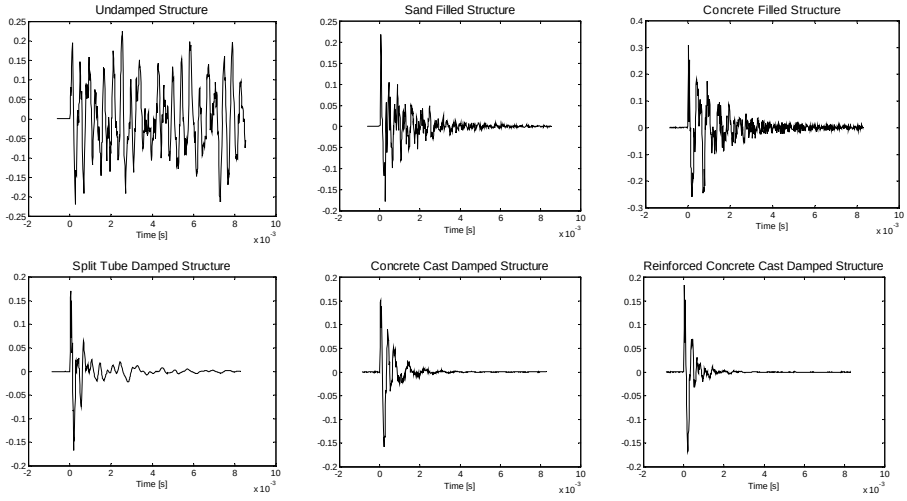


- Cable ties press damping sleeve against fixture
- Hot glue seals rebars

Test setup

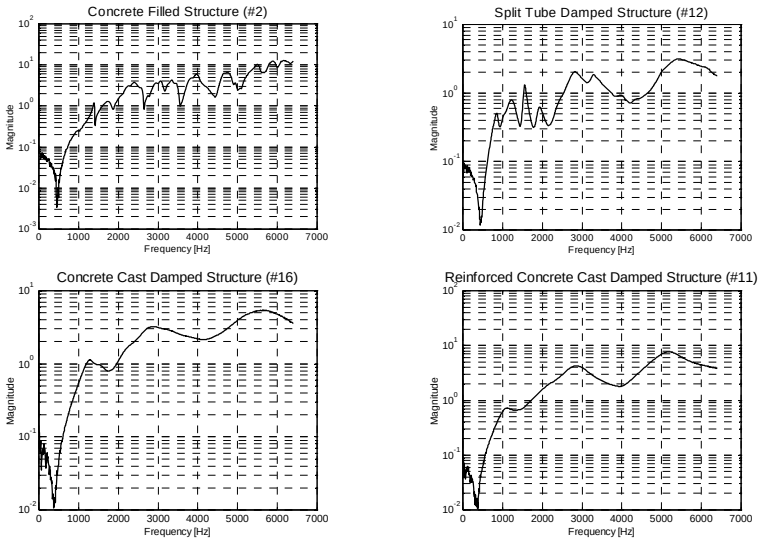
- HP 4-channel frequency analyzer
- 3-axis accelerometer
- 28 data points
- Free-free setup

Response to impulse



E. Bamberg, A.H. Slocum, "Concrete-based constrained layer damping", Precision Engineering, 26(4), October 2002, pp. 430-441.

Transfer Functions



Measured performance

- Split Tube
  - f=1530 Hz and η=0.055 (predicted: 0.032)
  - Material cost 100%
- Concrete Cast
  - f=1260 Hz and η=0.145 (predicted: 0.035)
  - Material cost 23.5%
- Reinforced Concrete Cast
  - f=1640 Hz and $\eta \approx 0.3$ (predicted: 0.044)
  - Material cost 28.8%

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Department of Mechanical Engineering
ME EN 7960 - Precision Machine Design

Problem Set 1 - Homogenous Transformation Matrices

Assigned: Wednesday, September 6, 2006

Due: Wednesday, September 13, 2006, 4 pm in 2110 MEB

Problem 1

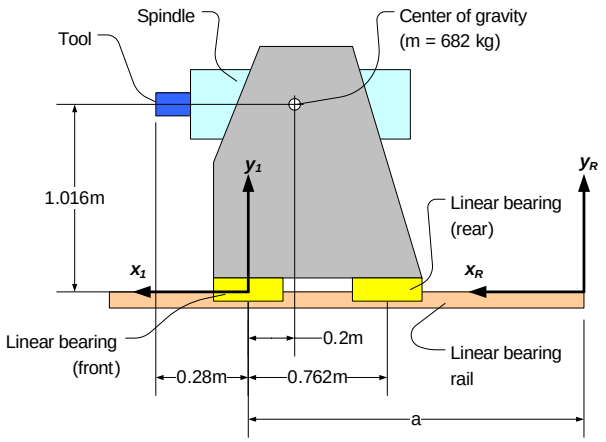

Figure 1 Linear axis

The linear axis shown in Figure 1 consists of two linear bearing trucks, a spindle with a mass *m* of 682 kg, and a carriage which can be treated as being weightless. The linear bearings can be treated as linear springs with a vertical stiffness of 350 E6 N/m.

a) Derive the homogenous transformation matrices required to determine the error that can expected at the tool tip (0.28m, 1.016m)? Determine the error at the tool tip as a result of gravity.

b) If the axis was calibrated after assembly to eliminate the effects of gravity but is now subject to a 0.5 g acceleration in the positive x-direction, what is the resulting error at the tool tip?

Problem 2

Figure 3 shows an assembly of two linear axes that are stacked up and perpendicular to each other, thereby providing the ability to position the work piece holder in the x-y plane of a machine tool. The y-axis is mounted on top of the x-axis. The offsets shown in Figure 3 show the starting positions of both axes.

a) For the 5 rail grades available, determine the maximum error (measured at the center of the work piece holder) as a result of geometric errors of the linear rails. Assume that the rails are straight (not bowed), but non-parallel. For the rail orientation for each axis refer to Figure 4.

Geometric details:

- The A axis coordinate system at the starting point is located at (550 mm, 150 mm, 450 mm) with respect to X_R,Y_R,Z_R.
- The B axis coordinate system at the starting point is located at (0 mm, 100 mm, -250 mm) with respect to X_A,Y_A,Z_A.
- The location of the work piece is (200 mm, 115 mm, 325 mm) with respect to X_B,Y_B,Z_B.

Problem 3

Assume that you are operating on a limited budget that allows you to only buy one pair of precision grade rails while the second pair would have to be a normal grade (regardless of length).

a) On which axis would you mount the precision grade and on which axis the normal grade?

b) What would be the difference in errors if you had the setup reversed? Determine the numerical value.

Problem 4

If the rails are not straight as in Problem 2 but bowed as shown in Figure 2, how would you have to modify the approach used in Problem 2 to arrive at a reasonable estimate in terms of error motions?

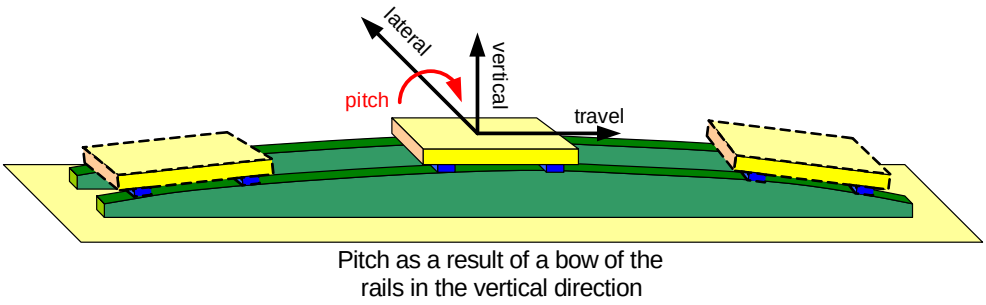

Figure 2 Non-straight bearing rails

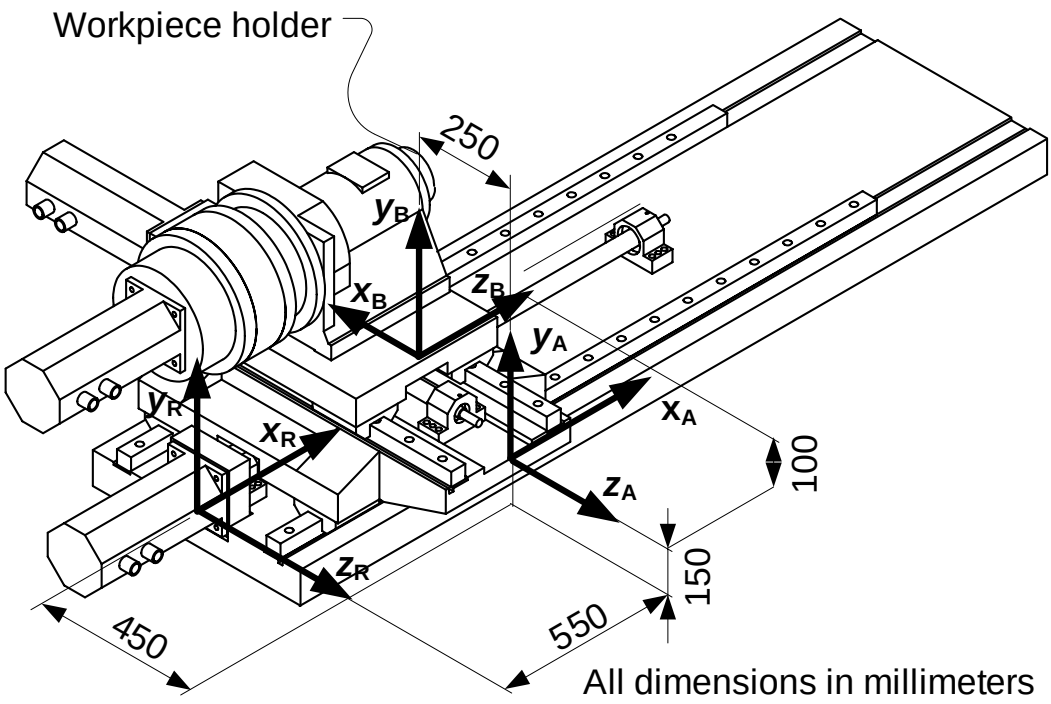

Figure 3 Stacked axes

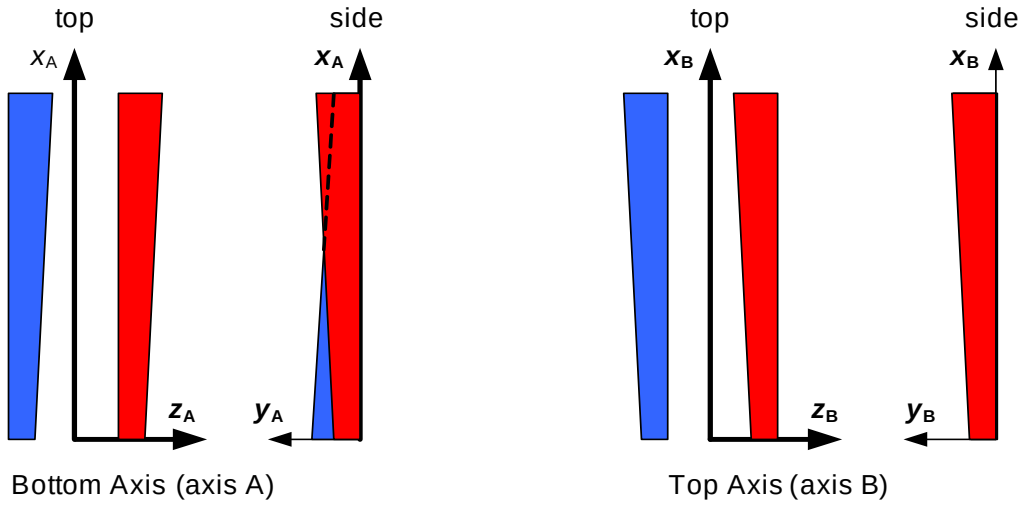

Figure 4 Arrangement of non-parallel rails

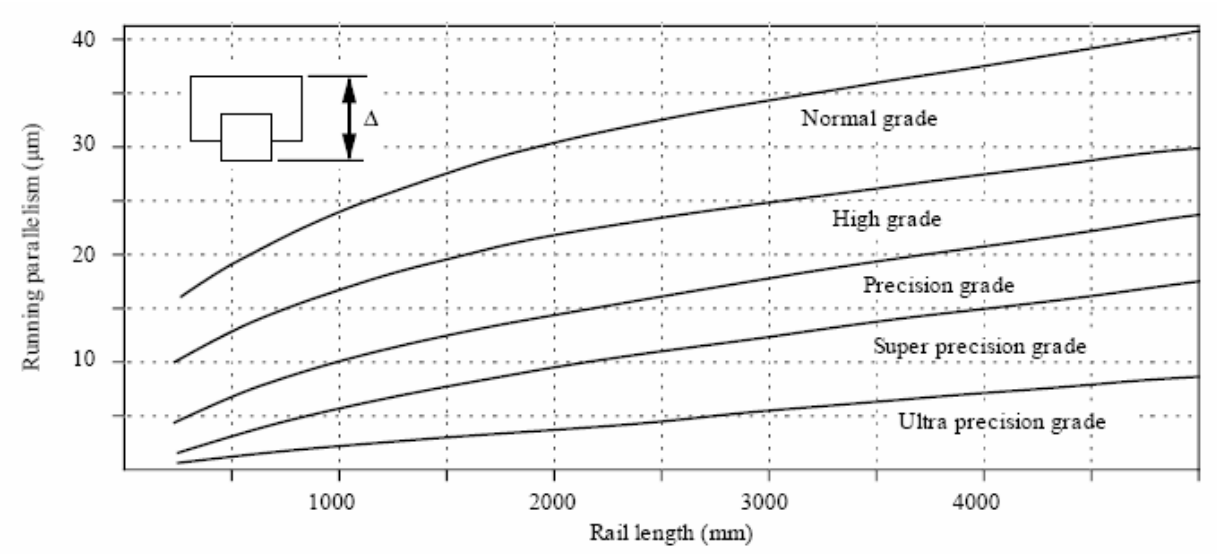

TABLE 1 Machine Data

Parameter	Value	Parameter	Value
axis A travel	1250 mm	axis B travel	350 mm
axis A rail spacing	350 mm	axis B rail spacing	250 mm
axis A bearing spacing	450 mm	axis B bearing spacing	300 mm

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Department of Mechanical Engineering
ME EN 7960 - Precision Machine Design

Problem Set 2 - Ball Screw Selection Tool

Assigned: Monday, October 9, 2006

Due: Monday, October 15, 2006, 4pm in 2110 MEB

Ballscrew Selection Tool

The selection of ballscrews is a multi-criteria problem that can be very time consuming and tedious. For this purpose, a tool that assists designers in making quick and reliable selections is an absolute must.

Problem 1

In this exercise you are asked to prepare an Excel spreadsheet that calculates all relevant selection criteria. Your spreadsheet should include the following THK ballscrew and nuts:

MDK Size 4 - 14, and BIF 16 - 50. The spreadsheet needs to have the following inputs:

Input Data

Parameter	Value	Units	Symbol	SI Value	SI Unit
Machining Data					
Machining force	650	N	F_m	650	N
Machining speed	0.3	m/min	v_m	0.005	m/s
Percentage of machining	75	percent	q_m	0.75	
Percentage of constant velocity	15	percent	q_uni	0.15	
Percentage of acceleration	10	percent	q_accl	0.1	
Rapid Motion	20	m/min	v_max	0.33333333	m/s
Acceleration time	0.1	s	t_ac	0.1	s
Table Data					
Mass	400	kg	m	400	kg
Incline angle	90	degrees	alpha	1.57079633	rad
Coefficient of friction	0.008		mu_bearing	0.008	
Length of ball screw	700	mm	L_axial	0.7	m
Gravity	9.81	m/s^2	g	9.81	m/s^2
Density	7850	kg/m^3	rho	7850	kg/m^3
Young's modulus	207	GPa	E	2.07E+11	Pa
Poison's ratio	0.3		poisons	0.3	
Shear modulus			G_modulus	7.96E+10	Pa
Ballscrew Data					
Mech. Efficiency	90	percent	eta	0.9	
Expected lifetime	40000	hr	life	40000	hr
Maximum allowable stress	147	Mpa	sigma_max	147000000	Pa
Safety Factors					
Safety against buckling	1.1		f_buckling	1.1	
Safety against brinelling	2		f_brinelling	2	
Safety against critical speed	1.1		f_critical_speed	1.1	
Safety against life	1.2		f_w	1.2	
Accuracy					
Rotary encoder resolution	4000	counts/rev	rot_resolution	4000	

The output should look similar to the one below.

MDK		
Type	**Unit**	**MDK 0401-3**
Motor		
Shaft speed during rapid motion	rpm	20000
Shaft speed during machining	rpm	300
Torque during rapid motion	Nm	0.93
Torque during machining	Nm	0.81
Ballscrew		
Equivalent stress < permissible stress?		No
Buckling load fixed-free > Fa?		No
Buckling load fixed-supp > Fa?		No
Buckling load fixed-fixed > Fa?		No
Safe against brinelling?		No
Critical speed > required speed fixed-free?		No
Critical speed > required speed fixed-supp?		No
Critical speed > required speed fixed-fixed?		No
DN value < 70000		No
Exceeds specified life?		No
Overall Stiffness		
Stiffness (fixed-free & fixed-supp)	N/micron	2.3
Stiffness (fixed-fixed)	N/micron	7.1
Accuracy		
Total error (fixed-free & fixed supp) with motor mount	micron	2090.3
Total error (fixed-fixed) with motor mount	micron	727.8
Total error (fixed-free & fixed supp) with free end mount	micron	2004.0
Total error (fixed-fixed) with free end mount	micron	641.5

Screw diameter	mm	4
Lead	mm	1
Root diameter	mm	3.4
Ballcircle diameter	mm	4.15
Dynamic load rating	kN	0.29
Static load rating	kN	0.42
Nut length	mm	13
Nut stiffness	N/micron	35
Support bearing stiffness	N/micron	27

Problem 2

Using the spreadsheet and the input parameters shown on the previous page, select the smallest (=least expensive) ballscrew that fulfills all relevant criteria as well as a required overall accuracy of 25 microns. What accuracy class will be required in this case?

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Department of Mechanical Engineering
ME EN 7960 - Precision Machine Design

Problem Set 3 - Linear Bearing Selection

Assigned:	Friday, October 27, 2006
Due:	Friday, November 10, 2006, 4 pm in MEB 2110

Problem 1

A common configuration for the design of linear axes is the use of rolling element linear guides which are available as off-the-shelf components in a number of different sizes from a number of different vendors.

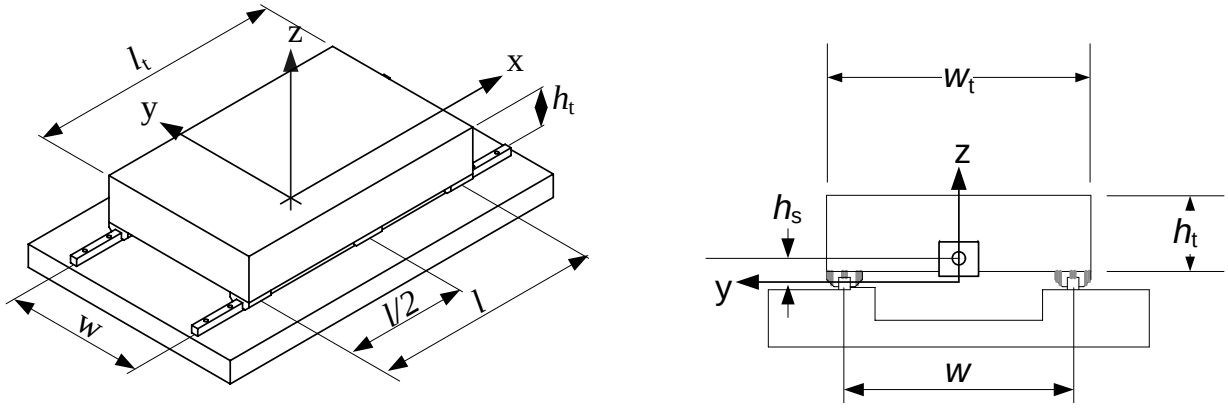

Figure 1 Linear axis consisting of 2 linear rails with 3 trucks each. The axis is driven by a ball screw.

a) Derive the stiffness matrix K (in analytical form) for the linear axis shown in Figure 1. Use the following assumptions:
   - You may assume that the axis consists of 6 identical linear bearing trucks whose lateral stiffness k_h and vertical stiffness k_v can be modeled as linear springs.
   - You may further assume that the table of this axis is significantly more rigid than the bearings, allowing you to model the table as a rigid body.
   - Include a ballscrew with rigidity k_s that is mounted along the x-axis at point $S(0,0,h_s)$.

b) Derive the mass matrix for this linear axis (in analytical form) and determine the natural frequencies of the systems.

Problem 2

To assist in the otherwise tedious process of bearing selection, you are asked to develop an electronic tool (e.g. Microsoft Excel) that contains a number of standard bearing sizes (see Table 1) and calculates all relevant parameters. For the linear axis shown in Figure 1, the tool needs to have the following abilities:

- Use 6 identical linear bearings that are arranged as shown in Figure 1 whose lateral and vertical stiffness are modeled using linear springs.

- Allow a 3D load be applied to the axis at any point.
- Determine the displacement of any point of the axis in 3D.
- Determine the stiffness of any point of the axis.
- Determine the loads onto the bearings.
- Determine the bearing life.
- Estimate the natural frequencies of such a system.
- Provide a boolean check of the bearing life based on the following input parameters: Machining speed, required lifetime.

TABLE 1 THK SR Linear Bearings

Size	Radial Dynamic Load Rating C [kN]	Radial Static Load Rating C_0 [kN]	Vertical Stiffness [N/μm]	Horizontal Stiffness [N/μm]
15	5.39	11.1	360	263
20	7.16	14.4	455	350
25	11.7	22.5	581	416
30	17.2	32.5	679	441
35	23.8	44.1	827	555
45	55.3	101	1125	737
55	89.1	157	1369	847

TABLE 2 THK SR Basic Load Ratings in Various Directions

Model No.	Direction	Basic Dynamic Load Rating	Basic Static Load Rating
SR 15 - 70	Radial (= vertical down)	C	C_0
	Reverse Radial (= vertical up)		$C_{0,RR} = 0.5C_0$
	Lateral		$C_{0,L} = 0.43C_0$

TABLE 3 THK SR Equivalent Factor

Model No.	P_E	*X*	*Y*
SR 15 - 70	Equivalent load in radial direction	1	1.155

Problem 3

Using the tool in Problem 2, analyze the following proposed design for the THK SR 35:

a) Provide the compliance matrix (in numerical form).

b) What are the displacements at the point of load application?

c) What is the stiffness in x, y, and z at the point of the load application?

d) What is the maximum equivalent load on the bearings?

e) What is the lifetime of the bearing that has the largest equivalent load? Use an application factor of 1.5.

f) What are the first three resonance frequencies?

g) Provide a printout of your electronic tool that answers the above listed questions.

TABLE 4 Data

Parameter	Value	Parameter	Value
Table length l_t	500 mm	**Bearing spacing l**	450 mm
Table width w_t	300 mm	**Rail spacing w**	250 mm
Table height h_t	300 mm	**Ballscrew rigidity k_s**	280 N/μm
Table density	7850 kg/m^3	**Ballscrew mount in x**	0 mm
Point of applied load in x	200 mm	**Ballscrew mount in y**	0 mm
Point of applied load in y	100 mm	**Ballscrew mount in z**	-30 mm
Point of applied load in z	300 mm	**Applied load in x**	1000 N
Machining speed	0.5 m/min	**Applied load in y**	1000 N
Required life	40,000 hrs	**Applied load in z**	1000 N